A Healthy Pet

A healthy diet for a dog includes protein, carbohydrates, fats, fiber, water, vitamins, and minerals. In general, a puppy needs about 100 Cal/lb (Calories per pound) of body weight daily, an adult dog needs about 60 Cal/lb, and a senior dog needs about 25 Cal/lb.



Data File Use the information on these two pages and on page 69 to answer these questions.

1. Research Use your own dog, or find information about a specific dog.

a. How much does the dog weigh?

b. Use the dog food label. Find the amount of food the dog will eat in a day.

c. One cup of dog food weighs about 8 oz. About how many meals will one bag contain?

d. About how much will it cost to feed the dog for a year?

e. Reasoning Which would cost more to feed for a year: two dogs this size or one dog that is twice as large?

2. A friend decides to start a dog-care business for people who travel.

a. Use the same dog as for Question 1. How much would it cost to feed the dog for two weeks?

b. Writing in Math Suppose the dog's owners are going on vacation for two weeks. Your friend decides to charge \$50/wk, which includes food, grooming, and walking the dog twice a day. Are the dog's owners likely to hire your friend? Why or why not?





\$369

Adult Dog Size	Daily Feeding
(pounds)	Dry (cups)
3–12 lb	1/2 to 1 1/4
13–20 lb	1 1/4 to 1 3/4
21-35 lb	1 3/4 to 2 2/3
36–50 lb	2 2/3 to 3 1/2
51-75 lb	3 1/2 to 4 3/4
76-100 lb	4 3/4 to 5 3/4
Over 100 lb	5 3/4 + 2/3 c for each 10 lb body weight over 100 lb



Take It to the NET For more information about dogs, go to www.PHSchool.com.

····· Web Code: abe-0253



Lessons

- 3-1 Exponents and Order of Operations
- 3-2 Scientific Notation
- 3-3 Divisibility Tests
- 3-4 Prime Factorization
- 3-5 Simplifying Fractions
- **3-6** Comparing and Ordering Fractions
- 3-7 Solve a Simpler Problem and Look for a Pattern
- 3-8 Mixed Numbers and Improper Fractions
- 3-9 Fractions and Decimals
- 3-10 Rational Numbers

Key Vocabulary

- composite number (p. 146)
- divisible (p. 141)
- equivalent fractions (p. 151)
- exponent (p. 131)
- greatest common factor (GCF) (p. 147)
- improper fraction (p. 164)
- least common denominator (LCD) (p. 156)
- least common multiple (LCM) (p. 145)
- mixed number (p. 164)
- prime number (p. 146)
- rational number (p. 173)
- repeating decimal (p. 169)
- scientific notation (p. 137)
- terminating decimal (p. 168)

Exponents, Factors, and Fractions





the camera. The rate at which a shutter opens and closes is called shutter speed.

A fast shutter speed means that the shutter opens and closes quickly. The shutter is open for a short amount of time and less light is allowed in. Fast-moving images require fast shutter speeds to prevent blurring.

Data File Shutter Opening Times (seconds)

You will use the data above in this chapter:

- p. 153
- Lesson 3-5
- p. 158
- Lesson 3-6
- p. 172
- Lesson 3-9



Real-World Snapshots On pages 182 and 183, you will solve problems involving photography.

Chapter 3 Preview

Where You've Been

- In Chapter 1, you performed operations with decimals and integers. You also learned how to use the Order of Operations.
- In Chapter 2, you learned to solve equations and inequalities.

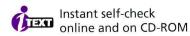
Where You're Going

 In Chapter 3, you will use the Order of Operations to simplify expressions with exponents. You will simplify, compare, and order fractions and decimals.



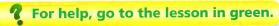
Scientists use exponents to express large measurements.

 Applying what you learn, you will use exponents to find the magnification of an image in a microscope.





Diagnosing Readiness



Adding and Subtracting Decimals (Lesson 1-2)

Find each sum or difference.

Multiplying and Dividing Integers (Lesson 1-8)

Find each product or quotient.

6.
$$-3 \cdot (-3)$$

7.
$$27 \div (-3)$$

6.
$$-3 \cdot (-3)$$
 7. $27 \div (-3)$ **8.** $-16 \div (-4)$

Order of Operations and the Distributive Property (Lesson 1-9)

Find the value of each expression.

9.
$$10 + 4(15 \div 5)$$

10.
$$30 \div 3 - 4 \cdot 2$$

10.
$$30 \div 3 - 4 \cdot 2$$
 11. $(8+4) \div 4 - 2$

Solving Two-Step Equations (Lesson 2-6)

Algebra Solve each equation.

12.
$$\frac{n}{8} + 5 = 4$$

13.
$$-5p - 3 = 22$$

14.
$$3x + 15 = 45$$

15.
$$-\frac{g}{7} - 6 = -4$$

16.
$$3a + 1 = -17$$

12.
$$\frac{n}{8} + 5 = 4$$
 13. $-5p - 3 = 22$ **14.** $3x + 15 = 45$ **15.** $-\frac{g}{7} - 6 = -4$ **16.** $3a + 1 = -17$ **17.** $-2 + \frac{b}{6} = -14$

Solving Inequalities by Multiplying or Dividing (Lesson 2-10)

Algebra Solve each inequality.

18.
$$4h \le -36$$

19.
$$-3t < 18$$

19.
$$-3t < 18$$
 20. $-\frac{m}{3} > 7$ **21.** $\frac{k}{2} \ge -5$

21.
$$\frac{k}{2} \ge -5$$



Exponents and Order of Operations

What You'll Learn



To write numbers with exponents



To simplify expressions with exponents using Order of Operations

... And Why

To find area in square miles, as in Example 2

Check Skills You'll Need

For help, go to Lesson 1-9.

Find the value of each expression.

1.
$$5 - 1 \cdot 3$$

2.
$$(5-1) \cdot 3$$

3.
$$4 \cdot (3+2)$$

4.
$$10 \div 2 - 3 \cdot 2$$

5.
$$10 + (2 + 4) \div 4$$

6.
$$(10+2) \div 4 + 5$$

7. Number Sense Consider any whole number greater than 2. Which produces a greater result, doubling it or multiplying it by itself?

New Vocabulary • exponent • power



OBJECTIVE



াত্রতা Interactive lesson includes instant self-check, tutorials, and activities.

Writing Numbers With Exponents



Suppose you're interviewing for an after-school job. The boss offers to pay you 2¢ the first day, with the amount to double each day. You investigate the offer using this table.

- 1. a. Complete the table to find how many cents you will be paid for each of the first five days.
 - b. How much will you be paid for the seventh day?
 - c. How much will you be paid for the tenth day?

Day	Cents
One	2 =
Two	2 × 2 = 🔳
Three	$2 \times 2 \times 2 = \square$
Four	$2 \times 2 \times 2 \times 2 = \blacksquare$
Five	$2 \times 2 \times 2 \times 2 \times 2 = \blacksquare$

2. Reasoning Suppose you are offered a different job that will pay you a \$10 flat fee for 10 days of work. Which job offer would you accept? Explain.

Reading Math

You read 5² as "5 to the second power," or "5 squared." You read 5³ as "5 to the third power," or "5 cubed."

You can use an exponent to show a product of equal factors. An **exponent** tells you how many times a number, or base, is used as a factor.

A number that can be expressed using an exponent is called a **power**. The number 125 is a power of 5 because it can be written as 5^3 .

Writing Expressions Using Exponents EXAMPLE

Write using an exponent.

a.
$$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$$

= $3^5 \leftarrow \frac{3}{5}$ is the base.
5 is the exponent.

b.
$$11 \cdot 11 \cdot 11$$

= $11^3 \leftarrow \frac{11}{3}$ is the base.

Calculator Hint

You can use the 🔼 or

If your calculator does

not have an exponent

4 🗶 4 🗶 4 🗏 64.

To find 4^3 , use 4 🔼 3 🗏 64 or

4 7 3 = 64.

y key to find a power.

- Check Understanding 1 a. Write $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ using an exponent.
 - **b.** Reasoning Suppose a is a nonzero number. How would you write $a \cdot a \cdot a$ using an exponent?

You can find the value of an expression with exponents by writing it as the product of repeated factors. You can also use a scientific calculator.

EXAMPLE

Real-World Problem Solving

Geography Gibraltar is at the western end of the Mediterranean Sea. Its area is about the same as the area of a square 1.5 mi on a side. Find Gibraltar's area.

Since $A = s^2$, find 1.5².

Method 1

$$1.5^2 = (1.5)(1.5) \leftarrow \text{Write as a product of repeated factors.}$$

= 2.25 $\leftarrow \text{Multiply.}$

Method 2

1.5
$$\mathbb{Z}^2 = 2.25 \leftarrow \text{Use the } \mathbb{Z}^2 \text{ key to square numbers.}$$

The area of Gibraltar is about 2.25 mi².



Check Understanding 2 Simplify. Use a calculator, paper and pencil, or mental math.

a. 3^5

b. 10^9

d. 1.2^3

OBJECTIVE

key, use

Simplifying Expressions With Exponents

To simplify expressions with exponents, you must consider exponents in the Order of Operations.

Key Concepts

Order of Operations

- **1.** Do all operations within groupings first.
- **2.** Evaluate any term(s) with exponents.
- 3. Multiply and divide in order from left to right.
- 4. Add and subtract in order from left to right.



Calculator Hint

To find $3^4 \cdot (7 - 2)^3$,



EXAMPLE

Simplifying Using Order of Operations

Simplify $3^4 \cdot (7 - 2)^3$.

$$3^4 \cdot (7-2)^3 = 3^4 \cdot 5^3 \leftarrow$$
 Do operations in parentheses.
= $81 \cdot 125 \leftarrow$ Find the values of the powers.
= $10{,}125 \leftarrow$ Multiply.



a.
$$(3^2 + 5) - 2$$

b.
$$3^2 + 5 - 2$$

c.
$$(3+5)^2-2$$

d. List the key strokes you use on your calculator to find $5 \cdot (9-2)^3$.

The expressions -5^4 and $(-5)^4$ are not equivalent. The expression -5^4 means the opposite, or the negative, of 5^4 . So the base of -5^4 is 5, not -5.

EXAMPLE

Simplifying Powers With Negatives

Simplify the expressions.

$$\mathbf{a.} - 5^4 = -1 \cdot 5^4$$
 \leftarrow Perform operations with exponents first.
 $= -(5 \cdot 5 \cdot 5 \cdot 5)$ \leftarrow The base is 5, not -5.
 $= -625$ \leftarrow Simplify.

b.
$$(-5)^4 = (-5)(-5)(-5)(-5)$$
 \leftarrow The base is the quantity in the parentheses, -5 . $= 625$ \leftarrow Multiply.



- **Check Understanding** 4 Simplify each expression.
 - **a.** $(-2)^3$ **b.** -2^3
- $\mathbf{c.} -3^4$
- **d.** $(-3)^4$
- e. Reasoning Does squaring a negative number always produce a positive result? Explain.

For more practice, see Extra Practice.



Write using an exponent.

Example 1 (page 132) **1.** 6 · 6 · 6

2. 2 · 2 · 2 · 2 · 2 · 2 · 2 · 3 **3.** 7 · 7 · 7 · 7

4. 8 · 8 · 8 · 8

5. 20 · 20 · 20 · 20 · 20 **6.** 12 · 12 · 12

Example 2 (page 132)

Simplify. Use a calculator, paper and pencil, or mental math.

7. 9²

8. 10⁸

9. 0.2^6

10. 1.7³

11. Each side of a sugar cube is approximately 0.6 in. long.

a. Calculator Find the volume of the sugar cube.

b. Which key strokes did you use?

Example 3 (page 133)

Simplify.

12.
$$2^3 \cdot (6-3)^2$$
 13. $(2^3 \cdot 6) - 3^2$

13.
$$(2^3 \cdot 6) - 3^2$$

14.
$$2^3 \cdot 6 - 3^2$$

15.
$$(2+6)^2 - 3^3$$
 16. $2^3 - 6 \cdot 3^2$ **17.** $(2^3 - 6) \cdot 3^2$

16.
$$2^3 - 6 \cdot 3^2$$

17.
$$(2^3-6)\cdot 3^2$$

18.
$$3^2 + 2^3 \cdot 6$$

19.
$$(3^2 + 2^3) \cdot 6$$

20.
$$3^2 \cdot (2^2 + 6)^2$$

Example 4 (page 133)

Simplify each expression.

21.
$$(-6)^3$$
 22. -2^4 **23.** $(-3)^7$ **24.** $(-4)^2$ **25.** -4^3

22.
$$-2^4$$

24.
$$(-4)^2$$

25.
$$-4^3$$



Use paper and pencil, mental math, or a calculator to simplify.

26.
$$2^5 \cdot 4^2$$

26.
$$2^5 \cdot 4^2$$
 27. $12 + 5^3$ **28.** -7^4

28.
$$-7^4$$

29.
$$8 + 3^4$$

30.
$$(-5)^5$$

31.
$$10^2 + 6^2$$

30.
$$(-5)^5$$
 31. $10^2 + 6^2$ **32.** $3(0.5 + 2.5)^2$ **33.** $(10 + 6)^2$

33.
$$(10+6)^2$$

34.
$$3(4^2-10)$$

34.
$$3(4^2-10)$$
 35. $(5-2^2)-1$ **36.** $4^3+14\div 7$ **37.** $(20\div 10)^2$

36.
$$4^3 + 14 \div 7$$

37.
$$(20 \div 10)$$



Real-World Connection



This dust mite has been magnified 1.5×10^5 times. 38. Technology A Scanning Electron Microscope (SEM) can magnify an image up to 10^5 times its size. How many times is this?

Science Match the fact with the power.

39. wheels on a unicycle

 $A. 2^5$

40. planets in the solar system

B. 3^2

41. freezing point of water in degrees Fahrenheit

C. 1^{17}

For Exercises 42–44, refer to the table.

- **42.** Copy the table. Fill in the missing values.
- **43.** Patterns What patterns do you notice?
- **44. Reasoning** Predict the number of zeros in 10^{12} .

Power of 10	Value	Number of Zeros
10^{1}	1981	1
10 ²	朣	1
10 ³	980	10
10 ⁴		
10 ⁵		

(Algebra) Evaluate each expression for m = 3, n = 2, and r = 4.

45.
$$m^2 + n^2$$

46.
$$(m+n)^2$$

47.
$$(-r)^3$$

48.
$$-m^3$$

49.
$$n^2(m+r)$$

50.
$$(n^2r^2-m)^2$$

- **51. a.** What exponent completes the table below?
- **b.** Calculator Find $2^0, 3^0, 4^0, 5^0$, and 10^0 .

Value	16	8	4	2	1
Power of 2	24	23	22	2^{1}	2

52. Writing in Math Write a general rule to find the value of a nonzero expression with an exponent of 0.

- 53. a. Geometry How many small squares line up on one edge of the larger square?
 - **b.** How many small squares make up the whole square?
 - **c.** Reasoning Why do you think 5^2 is referred to as "5 squared"?



54. Estimation Write two powers that are between the values of 8^2 and 8^3 .



Algebra Evaluate each expression for f = -2, g = -3, and h = -4.

55.
$$(f+g)^2$$

56.
$$(-h)^3$$

57.
$$-f^3$$

56.
$$(-h)^3$$
 57. $-f^3$ **58.** $g^2(f+h)$

59. Stretch Your Thinking Paulette has six marbles that are the same size. Five of her marbles weigh the same. The sixth marble is heavier than the rest. Hahn is trying to find which marble is heavier than the rest by using a balance scale and only two weighings. How can he do this?



Test Prep

Multiple Choice

- **60.** Evaluate the expression $3^3 24 \div 6$.
 - A. 0.5
- **C**. 21
- D. 23
- **61.** On a test, the class average is 3⁴. Your score is 6 points above the class average. What is your score on the test?
 - **F.** 75
- **G.** 81
- I. 92
- **62.** Order from least to greatest: 4^3 , 9^2 , 2^7 , $(-5)^2$.

A.
$$2^7$$
, 4^3 , $(-5)^2$, 9^2

B.
$$(-5)^2$$
, 4^3 , 9^2 , 2^7

C.
$$9^2$$
, $(-5)^2$, 2^7 , 4^3

D.
$$9^2$$
, $(-5)^2$, 4^3 , 2^7

Extended Response

63. Your friend sends you the following message written in a secret code.

$$-8 -1$$

Each number in the code represents a letter, as follows:

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1	Online lesson quiz at
	www.PHSchool.com
	Web Code: aba-0301

Letter
 A
 E
 L
 M
 N
 R
 S
 T

 Power

$$1^3$$
 $(-1)^3$
 2^3
 $(-2)^3$
 3^3
 $(-3)^3$
 4^2
 5^2

What does your friend's message say?

Mixed Review

Algebra Solve each inequality. Lesson 2-10

64.
$$\frac{a}{2} > 10$$

65.
$$-3n < -9$$
 66. $\frac{p}{-6} \le 5$ **67.** $\frac{z}{12} \ge -1$

66.
$$\frac{p}{-6} \le 3$$

67.
$$\frac{z}{12} \ge -1$$

Lesson 1-8 Find the value of each expression.

70.
$$-8 \div (-4)$$

69.
$$-4(-10)$$
 70. $-8 \div (-4)$ **71.** $49 \div (-7)$



Scientific Notation

What You'll Learn



To write numbers in scientific notation



To write numbers in standard form

... And Why

To express the moon's distance from Earth, as in Example 1

Need Help?

For practice with powers

page 706.

of 10, see Skills Handbook

✓ Check Skills You'll Need

For help, go to Lesson 3-1.

Simplify.

1. 3³

2. 4²

3. 10^5

4. 1¹⁵

5. 10¹

- **6.** 2⁸
- 7. Number Sense Is 10^7 closer to a million or a billion? Answer without calculating.

OBJECTIVE



Interactive lesson includes instant self-check, tutorials, and activities.

Writing Numbers in Scientific Notation

Investigation: Powers of 10

1. Copy and complete the table below.

Factors	10 · 1	10 · 10	10 · 10 · 10	10 · 10 · 10 · 10
Product	10	100	- 18	R R
Number of Zeros in Product	1			

- 2. Study the pattern in your table. How is the number of zeros in the product related to the number of times 10 is used as a factor?
 - 3. How many times should 10 be used as a factor to obtain 100,000,000,000?
 - **4. Calculator** Copy the table below. Use the pattern from the table above to complete the missing information in the table below.

Factors	$3.5 \times 1,000$	$7.2 \times 10,000$		$36.8 \times 1,000,000,000$
Product	3,500	12	4,800,000	

5. Different calculators have different ways of showing scientific notation. How does your calculator display the result in the last row of the last column of this table?

Scientific notation is a shorthand way to write numbers using powers of 10.

Key Concepts

Scientific Notation

A number in scientific notation is written as the product of two factors, one greater than or equal to 1 and less than 10, and the other a power of 10.

$$10^{12}$$

Second factor is a power of 10.

First factor is greater than or equal to 1, but less than 10.



Calculator Hint

On calculators that do not display scientific notation, powers of 10 that are too large for the screen are shown with the letter "E." 2 E12 means 2×10^{12} .



Writing in Scientific Notation



Science The moon orbits Earth at a distance of 384,000 km from Earth. Write this number in scientific notation.

3.84000, \leftarrow Move the decimal point to get a factor greater than 1 but less than 10.

$$384,000 = 3.84 \times 100,000 \qquad \leftarrow \text{Write as a product of 2 factors.} \\ = 3.84 \times 10^5 \qquad \leftarrow \text{Write 100,000 as a power of 10.}$$

 $\stackrel{\bullet}{\bullet}$ The moon orbits Earth at a distance of 3.84 \times 10⁵ km.



- Check Understanding 1 a. Write 396,000,000 in scientific notation.
 - **b.** Reasoning Is 107×10^4 written in scientific notation? Explain.



Writing Numbers in Standard Form

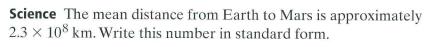
You can change expressions from scientific notation to standard form by simplifying the product of the two factors.



EXAMPLE

Writing in Standard Form







Earth from space

Method 1

$$2.3 \times 10^8 = 2.3 \times 100,000,000$$
 \leftarrow Write as a product of 2 factors.

= 230,000,000← Multiply the factors.

Method 2

$$2.3 \times 10^8 = 2.30000000$$

= 230,000,000
The exponent is 8. Move the decimal 8 places to the right.

The mean distance is approximately 230,000,000 km.





a.
$$1.2 \times 10^2$$

b.
$$3.32 \times 10^5$$

c.
$$6.443 \times 10^9$$



Write in scientific notation. Exercise 1 has been started for you.

Example 1 (page 137)

1.
$$7,500$$

= 7.5×10^{-1}

Example 2 (page 137)

Write in standard form.

9.
$$3.4 \times 10^3$$

10.
$$5.9 \times 10^2$$

11.
$$8.21 \times 10^3$$

12.
$$6.678 \times 10^2$$

13.
$$7.45 \times 10^4$$

14.
$$9.9673 \times 10^2$$

15.
$$5 \times 10^{11}$$

16.
$$7.02 \times 10^{1}$$

17.
$$6.15 \times 10^8$$

18.
$$2.439 \times 10^7$$

19.
$$4.77 \times 10^9$$

20.
$$2.25 \times 10^3$$

Apply Your Skills

Explain why each number is not in scientific notation.

21.
$$35.4 \times 10^6$$

22.
$$8.63 \times 2^{10}$$

23.
$$0.387 \times 10^7$$

24.
$$75.5 \times 10^7$$

25. Science The table shows the planets in the solar system and their masses. List their masses in order from least to greatest.

For Exercises 26-31, write each number in scientific notation.

- **26. Plants** There are about 350,000 different kinds of plants on Earth.
- 27. Geography The Folsom Dam in California holds back 326 billion gallons of water.

28.
$$34.5 \times 10^3$$

29.
$$1,228 \times 10^2$$

30.
$$122.85 \times 10^2$$

31.
$$312 \times 10^6$$

Mass (kg) **Planet** 3.303×10^{23} Mercury 4.869×10^{24} Venus 5.976×10^{24} Earth 6.421×10^{23} Mars 1.900×10^{27} Jupiter 5.688×10^{26} Saturn 8.686×10^{25} Uranus 1.024×10^{26} Neptune 1.290×10^{22} Pluto

Math in the Media Use the cartoon below for Exercises 32–33.

FOX TROT

by Bill Amend



- **32.** How many minutes was the warning? Write in standard form.
- 33. Convert the time to hours. Write in scientific notation.



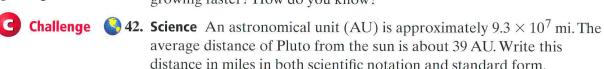
Real-World Connection

This balloon's basket is made of wicker, which is sturdy but lightweight.



For Exercises 34–39, write each number in standard form.

- **34.** 0.345×10^3
- 35. 1.203×10^{15}
- **36.** $0.000000312 \times 10^{10}$
- \bigcirc 37. Ballooning The first balloon to carry passengers weighed 1.6×10^3 lb.
- \bigcirc 38. Light One light-year is 5.88×10^{12} mi.
- **Science** There are about 2×10^{13} red blood cells in a 125-lb person.
 - 40. Writing in Math Explain how you would find the power of 10 to write 725,000,000 in scientific notation.
- 41. Biology In a laboratory experiment, two colonies of bacteria are being observed. The first is growing at a rate of 2.2×10^6 bacteria per hour. The other is growing at a rate of 6.3×10^5 bacteria per hour. Which is growing faster? How do you know?



43. Stretch Your Thinking For every two goldfish you buy at the regular price, you get a third goldfish for a penny. You spend \$.45 for nine goldfish. Find, in cents, the regular price of one goldfish.



Test Prep

Multiple Choice

44. How is 30,240,000,000 written in scientific notation?

- **A.** 30.24×10^9 **B.** 30.24×10^6 **C.** 3.024×10^{11} **D.** 3.024×10^{10}
- ake It to the NET Online lesson quiz at www.PHSchool.com Web Code: aba-0302
- **45.** How is 6.023×10^4 written in standard notation? **F.** 60.23 **G.** 602.3
- H. 6,023
- I. 60,230
- 46. Which of the following is NOT written in scientific notation? **B.** 2.0×10^2 **C.** 3.24×10^3
 - **A.** 4.127×10^{1}

Short Response

- 47. A penny with a diameter of 0.75 in. is magnified to 100,000 times its size in an electron microscope.
 - a. Write the size that the image would be in scientific notation.
 - **b.** Write the size that the image would be in standard form.

Mixed Review

Lesson 3-1 Simplify.

48.
$$4^2 + 2^3 \cdot 3$$

49.
$$(4^2 + 2^3) \cdot 3$$

50.
$$4^2 \cdot (2^3 + 3)^2$$

Lesson 2-9

Algebra Solve each inequality.

51.
$$-5 > c - 7$$

52.
$$-56 \le -7 + y$$

53.
$$14 + w \ge -4$$

Negative Exponents

For Use With Lesson 3-2

Remember that a number in scientific notation is written as a product of two factors, one greater than or equal to 1 and less than 10, and the other a power of 10. To write a number between 0 and 1 in scientific notation, you can use a negative exponent.

Writing in Scientific Notation EXAMPLE

Write 0.0084 in scientific notation.

0.008.4

← Move the decimal point to obtain a factor greater than 1 but less than 10.

 $0.0084 = 8.4 \times 0.001 \leftarrow \text{Write as a product of 2 factors.}$

 $= 8.4 \times 10^{-3} \quad \leftarrow$ The decimal point was moved 3 places to the right. Use -3 as the exponent.

In scientific notation, 0.0084 is written as 8.4×10^{-3} .

EXAMPLE Writing in Standard Form

Write 3.52×10^{-5} in standard form.

Method 1

 $3.52 \times 10^{-5} = 3.52 \times 0.00001$ \leftarrow Write as a product of 2 factors. = 0.0000352← Multiply the factors.

Method 2

 $3.52 \times 10^{-5} = 0.00003.52$ \leftarrow The exponent of 10 is -5. Move the decimal 5 places to the left. = 0.0000352

 $\frac{1}{6}$ The value of 3.52×10^{-5} is 0.0000352.

EXERCISES

Write each number in scientific notation.

1. 0.0008

2. 0.037

3. 0.0000422

4. 0.00000691

5. 0.005006

6. 0.00147

7. 0.5

8. 0.049562

Write each number in standard form.

9. 2.8×10^{-3}

10. 8.55×10^{-1}

11. 8.33×10^{-6}

12. 1.381×10^{-4}

13. 2.005×10^{-2}

14. 6.079×10^{-5}

15. The width of a hair is about 3×10^{-7} in.

16. A flea weighs 4.9×10^{-3} g.



Divisibility Tests

What You'll Learn



To use divisibility

... And Why

To solve problems involving codes, as in Example 3

Check Skills You'll Need

For help, go to Lesson 1-8.

Simplify.

2.
$$-27 \div (-9)$$

4.
$$-150 \div 10$$

New Vocabulary • divisible





INTERACTIVE lesson includes instant self-check, tutorials, and activities.

Using Divisibility Tests

One whole number is **divisible** by a second whole number if the remainder is 0 when you divide the first number by the second number. Since $16 \div 2 = 8, 16$ is divisible by 2.

Key Concepts

Divisibility Tests for 2, 4, 5, 8, and 10

A whole number is divisible by

- 2 if it ends in 0, 2, 4, 6, or 8.
- 4 if the number formed by the last two digits is divisible by 4.
- 5 if it ends in 0 or 5.
- 8 if the number formed by the last three digits is divisible by 8.
- 10 if it ends in 0.

EXAMPLE

Divisibility by 2, 4, 5, 8, and 10

Is the first number divisible by the second? Explain.

- **a.** 567 by 2
- No, 567 does not end in 0, 2, 4, 6, or 8.
- **b.** 1,015 by 5
- Yes, 1,015 ends in 5.
- **c.** 111,120 by 10
- Yes, 111,120 ends in 0.
- **d.** 934 by 4
- No, 34 is not divisible by 4.
- e. 29,640 by 8
- Yes, 640 is divisible by 8.

- **Check Understanding** 1 Is the first number divisible by the second? Explain.
 - **a.** 160 by 5
- **b.** 76 by 10
- **c.** 33,560 by 4
- **d.** 1,856 by 8
- e. Reasoning Is any number that is divisible by both 2 and 5 also divisible by 10? Explain.

The table below shows a pattern for divisibility by 3 and 9.

		divis	e sum ible by	divisi	number ble by
Number	Sum of Digits	3?	9?	3?	9?
215	2+1+5=8	No	No	No	No
282	2 + 8 + 2 = 12	Yes	No	Yes	No
468	4 + 6 + 8 = 18	Yes	Yes	Yes	Yes

Key Concepts

Divisibility Tests for 3 and 9

A whole number is divisible by

- 3 if the sum of its digits is divisible by 3.
- 9 if the sum of its digits is divisible by 9.

EXAMPLE

Divisibility by 3 and 9

Is the first number divisible by the second? Explain.

Yes,
$$4 + 6 + 5 = 15$$
, which is divisible by 3.

No,
$$3 + 0 + 1 + 6 = 10$$
, which is not divisible by 9.



Check Understanding 2 Is the first number divisible by the second? Explain.

a. 262 by 3

b. 13,449 by 3

c. 586 by 9

d. 30,756 by 9



Use a combination of tests to find the factors by which a number is divisible.

EXAMPLE

Real-World Problem Solving

Codes A friend sends you a message signed with the code number 5,385. Dave's number is divisible by 3, 5, and 8. Janice's number is divisible by 2 and 3, but not 4. Joshua's number is divisible by 4 and 5, but not 3. Karen's number is divisible by 3 and 5, but not 8. Who sent the message?

Use logical reasoning to see which friend's divisibility rules match the divisibility rules for 5,385.

Is 5,385 divisible by 2?

No, it does not end in 0, 2, 4, 6, or 8.

Is 5,385 divisible by 3?

Yes, 5 + 3 + 8 + 5 = 21, which is divisible by 3.

Is 5,385 divisible by 4?

No, 85 is not divisible by 4.

Is 5,385 divisible by 5?

Yes, it ends in 5.

Is 5,385 divisible by 8?

No, 385 is not divisible by 8.

5,385 is divisible by 3 and 5, but not 8, so the message is from Karen.



Real-World Connection

Ancient Egyptians used

codes to represent measurements and other

numerical values.

Check Understanding 3 A message is signed with the number 32,680. Who wrote it?

EXERCISES



Is the first number divisible by the second? Explain.

Example 1 (page 141)

- 1. 571 by 2
- **2.** 5,605 by 10
- **3.** 3,650 by 5

- **4.** 3,179,144 by 4
- **5.** 82,240 by 8
- **6.** 6,722,540 by 10

- **7.** 6,790 by 2
- **8.** 3,491,660 by 4
- **9.** 110,344 by 8

Example 2 (page 142)

- **10.** 558 by 3
- **11.** 675 by 9
- **12.** 8,394 by 3

- **13.** 2,472 by 9
- **14.** 7,623,540 by 3
- **15.** 7,623,540 by 9

Example 3 (page 142)

- **16. Fashion** You direct marketing for a fashion magazine. To distinguish your magazine from others, you identify it with a four-digit code. The number formed by the last three digits of the code is divisible by the first digit. Determine whether each number can be used.
 - **a.** 4,956
- **b.** 5,625
- **c.** 9.585
- **d.** 3,173
- 17. Games In an adventure game you must correctly place three colored keys into a door to gain access to a treasure room. Each key has a number etched into it. Use the clues to find the correct keys to use.



- The top lock uses a key that is divisible by 3 and 4, but not 5.
- The middle lock uses a key that is divisible by 2 and 5, but not 3.
- The bottom lock uses a key that is divisible by 3 and 5, but not 4.

Apply Your Skills

Tell whether each number is divisible by 2, 3, 4, 5, 8, 9, or 10. Some numbers may be divisible by more than one number.

- **18.** 324
- **19.** 150
- **20.** 840
- **21.** 2,724
- **22.** 1,430

- **23.** 6,720
- **24.** 81.816
- **25.** 625
- **26.** 7,848
- **27.** 4,725
- **28. Shipping** A company wants to ship 63,720 model airplanes. The size of a shipping carton is measured by the number of models it holds. Each carton must hold more than 3 but less than 10 models. Which sizes can the company use? How many cartons are needed for each size?
 - **29.** Refer to these numbers: 10 66 898
- 975 4,710
- **a.** Which are divisible by both 2 and 3?
- **b.** Which are divisible by 6?
- **c.** Using your results, write a divisibility test for 6.
- **30.** Writing in Math Write divisibility tests for 12 and 15.

Number Sense If you scramble the digits of a number that is divisible by 3, it is still divisible by 3. Does this work for the numbers below? Explain.

- **31.** numbers divisible by 2
- **32.** numbers divisible by 4
- **33.** numbers divisible by 8
- **34.** numbers divisible by 9

Write the missing digit to make each number divisible by 9.

- **35.** 22 ,043
- **36.** 3 , 187
- **37.** 2,03 ,371
- **38.** 1 ,012



- **39.** Here is the divisibility rule for 11. Calculate the sum of every other digit. Then find the sum of the remaining digits of the number. Find the difference between the sums. If the result is a multiple of 11, then the number is a multiple of 11. Tell whether each is a multiple of 11.
 - **a.** 3.577
- **b.** 4,818
- **c.** 32,417
- **d.** 361,526
- **40.** A number is greater than 500 and less than 550. The number is a multiple of 9 and the units digit is 1. What is the number?
- **41. Stretch Your Thinking** You work at a dog kennel. You use 5 lb of dog food to feed 3 dogs for 4 days. At the same feeding rate, how many pounds of dog food will you need to feed 12 dogs for 1 week?



Test Prep

Multiple Choice

- **42.** Which number is divisible by 3?
 - **A.** 18,073
- **B.** 20,412
- **C**. 28,412
- **D.** 31,415

- 43. Which number is NOT divisible by 9?
 - **F.** 1,269
- **G**. 2,195
- **H.** 3,150
- I. 4,761



- 44. Which number is divisible by 4?
 - **A.** 12,542
- **B.** 12,524
- **C.** 12,254
- **D.** 12,245
- 45. Eight photo albums contain an equal number of photos. Which could be the total number of photos?
 - F. 1,821
- **G.** 1,218
- **H.** 1,182
- I. 1,128

Mixed Review

Write each number in scientific notation. Lesson 3-2

- **46.** 5,200,000
- **47.** 348,000
- **48.** 7,100

- **49.** 6,125
- **50.** 8,901,067,000
- **51.** 123,456,789

Lesson 2-6

Algebra Solve each equation.

- **52.** -3y 10 = 17 **53.** $\frac{x}{4} + 1 = 13$ **54.** $\frac{t}{-7} + 5 = 6$



Prime Factorization

What You'll Learn



To find multiples and factors



To use prime factorization

... And Why

To determine computer delivery schedules, as in Example 1

✓ Check Skills You'll Need

For help, go to Lesson 3-3.

Tell whether the first number is divisible by the second number. Explain.

1. 48; 2

2. 48: 3

3. 48; 4

4. 48; 5

5. 48: 6

6. 48: 8

New Vocabulary • multiple • least common multiple (LCM) • factor composite number
 prime number
 prime factorization • greatest common factor (GCF)





Interactive lesson includes instant self-check, tutorials, and activities.

Finding Multiples and Factors

Suppose you and a friend volunteer at an animal shelter. Today you are both there. You volunteer every third day and your friend volunteers every fourth day. You will see each other again 12 days from today.



Real-World Connection

Animal shelter workers rescued this baby possum from a car crash.



The diagram above shows multiples of 3 and 4. A multiple of a number is the product of that number and any nonzero whole number. The least **common multiple (LCM)** of two or more numbers is the least multiple that is common to all of the numbers. For example, the LCM of 3 and 4 is 12.

EXAMPLE

Finding the Least Common Multiple



Business Computers are delivered to a store every 6 days. Printers are delivered every 9 days. If both printers and computers are delivered today, when will both be delivered again?

Find the least common multiple of 6 and 9.

Multiples of 6: 6, 12, 18, 24, 30, 36, ... Multiples of 9: 9, 18, 27, 36, ...

The LCM of 6 and 9 is 18. So, both will be delivered in 18 days.



- \checkmark Check Understanding $extbf{1}$ Find the LCM of each pair of numbers.
 - **a.** 4, 10

b. 5, 7

- **c.** 12, 15
- **d.** Reasoning Zero is not considered a multiple of any number other than itself. Why does this make sense?

A factor is a whole number that divides another whole number with a remainder of 0. Any number is always divisible by all of its factors.

EXAMPLE Finding Factors

Find the factors of 30.

 $2 \cdot 15$ $3 \cdot 10$ $5 \cdot 6$ \leftarrow Find pairs of numbers with a product of 30.

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.



- Check Understanding 2 a. Find the factors of 42.
 - **b.** Number Sense What numbers are always factors of even numbers?

OBJECTIVE

Using Prime Factorization

A composite number is a whole number greater than 1 that has more than two factors. A prime number is a whole number with exactly two factors, 1 and the number itself. The number 1 is neither prime nor composite.

EXAMPLE Prime Numbers and Composite Numbers

Tell whether each number is prime or composite.

Factors: 1, 2, 3, 4, 6, 12 **a.** 12

12 is a composite number.

b. 13 Factors: 1, 13 13 is a prime number.



- Check Understanding 3 Tell whether each number is prime or composite. **b.** 15
 - **a.** 14

- **d.** 2
- **e.** Reasoning What is the only even prime number?

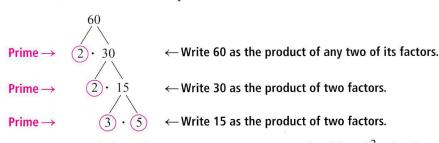
Writing a composite number as the product of its prime factors shows its **prime factorization.** You can use a *factor tree* to find the prime factors.

EXAMPLE **Writing Prime Factorization**

Use a factor tree to write the prime factorization of 60.

Need Help?

For help with writing expressions involving exponents, go to Lesson 3-1.



 $60 = 2 \cdot 2 \cdot 3 \cdot 5$. Or, using exponents, you can write $60 = 2^2 \cdot 3 \cdot 5$.



Check Understanding 4 Write the prime factorization of 72. Use exponents where possible.

The greatest common factor (GCF) of two or more numbers is the greatest number that is a factor of all the numbers.

EXAMPLE **Finding the Greatest Common Factor**

Find the GCF of 24 and 36.

$$24 = 2 \cdot 2 \cdot 2 \cdot 3$$
 $36 = 2 \cdot 2 \cdot 3 \cdot 3$ \leftarrow Write the prime factorizations.

$$GCF = 2 \cdot 2 \cdot 3 = 12$$
 \leftarrow Find the product of the common factors.

The GCF of 24 and 36 is 12.

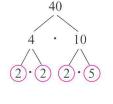


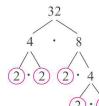
More Than One Way

Two gymnastics teams are marching at an event. There are 32 members on one team and 40 on the other. They are marching in rows of equal size that are as wide as possible. How many people are in each row?

Carlos's Method

First, I'll write the prime factorizations of 40 and 32. Then I'll find the GCF.





← Write as products of two factors.

← Write as products of two factors.

← Write as a product of two factors.

 $40 = 2 \cdot 2 \cdot 2 \cdot 5$ $32 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \leftarrow$ Write the prime factorizations. $GCF = 2 \cdot 2 \cdot 2 = 8$ \leftarrow Find the product of the common factors.

The teams should march in rows of 8 people.

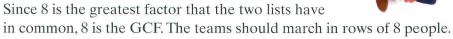


First I'll list the possible sizes of rows.

Then I'll choose the largest number in both teams.

The factors of 32 are: 1, 2, 4, 8, 16, 32

The factors of 40 are: 1, 2, 4, 5, 8, 10, 20, 40



Choose a Method

Teams of 36 and 60 are to march in rows of equal width. How wide is the widest row possible? Describe your method and explain why you chose it.





Find the LCM of each pair of numbers.

Example 1 (page 145)

1. 4, 6

2. 9, 12

3. 8, 5

4. 2, 5

5. 3, 8

6. 6, 7

7. 5. 10

8. 10, 6

9. 15, 9

10. 24, 8

11. Fitness Suppose you take aerobics classes every 3 days and martial arts classes every 7 days. If you have both classes today, when will you take both classes again on the same day?

Example 2 (page 146)

Find the factors of each number.

12. 20

13. 23

14. 32

15. 62

16. 70

17. 36

18. 40

19. 50

20. 44

21. 63

Example 3 (page 146)

Tell whether each number is prime or composite.

22. 7

23. 16

24. 21

25. 1

26. 6

Example 4 (page 146)

Write the prime factorization. Use exponents where possible.

27. 45

28. 64

29. 84

30. 111

31. 65

32. 52

33. 75

34. 48

35. 60

36. 132

Example 5 (page 147)

Find the GCF of each pair of numbers.

37. 18, 32

38. 12, 15

39. 16, 80

40. 10, 85

41. 38, 76

42. 75, 90

43. 54, 80

44. 52, 26

B Apply Your Skills

Tell whether each number is prime or composite.

45. 47

46. 69

47. 77

48. 165

49. 104

50. Error Analysis Two students made factor trees of the prime factors of 24. Are both correct? Explain.

4 · 6 / (2) · (2) · (3)

Sal

24

Marla 24 /\ (3) \cdot 8

51. Writing in Math Describe the relationships between 15, 5, and 3 using the words *factor* and *multiple*.

52. Reasoning Find two composite numbers with a GCF of 1.

Write the prime factorization. Use exponents where possible.

53. 86

54. 144

55. 210

56. 675

57. 720

58. 1,000

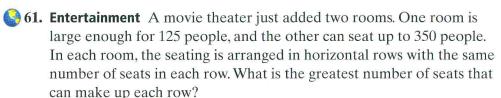
59. 340

60. 1,234



Real-World (Connection

Theater seats are staggered so people in each row do not block the views of people behind them.



Mental Math Find the GCF of each pair of numbers.

- **62.** 3, 10
- **63.** 7, 12
- **64.** 4, 20
- **65.** 50, 1000
- **66.** Reasoning You can express the number 100 as 10^2 using exponents. Is this the same as the prime factorization of 100? Explain.
- **67.** The diagram shows the prime factors of 18 and 24.
 - a. Find the product of the factors that are in the intersection of the circles. Is the product the GCF or the LCM? Explain.
- 18 24 3 2 3 2 2
- **b.** Find the product of all the factors in both circles. Is the product the GCF or the LCM? Explain.
- **68.** Number Sense Let *n* be any prime number. Tell whether each statement is sometimes, always, or never true.
 - **a.** 2*n* is composite.

b. 2n + 1 is prime.

c. 2n + 1 is even.

- **d.** 2n + 2 is composite.
- **69. Stretch Your Thinking** A number is divided by 6. The quotient is doubled and then 8 is added. The answer is 24. What is the number?



Test Prep

Challenge

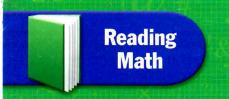
- **Gridded Response**
- 70. Find the GCF of 21 and 15.
- 71. Find the GCF of 32 and 24.
- 72. Find the LCM of 12 and 8.
- 73. Find the LCM of 9 and 1.
- 74. What number is the greatest prime factor of 308?



Mixed Review

Find the value of each expression. Lesson 1-7

Order from least to greatest. Lesson 1-6



Reading a Math Lesson

For Use With Lesson 3-4

When you read a math lesson, keep the purpose in mind. Here are some strategies you can use to help you read for the main purpose in the lesson.

Focus Before you read a lesson:

- Look at the title.
- Find the objectives.
- For each objective, write a question.

Objectives show you what is important. The questions you write can help you remember the information you are trying to find and learn.

Lesson 3-4 Prime Factorization

Objective	Questions for the Objective
1. Finding Multiples and	a. What is a multiple?
Factors	b. What is a factor?
	c. How are they similar?
	d. How are they different?
2. Using Prime Factorization	e. What is prime factorization?f. Why is prime factorization useful?

Read As you read:

- Try to answer your questions.
- If necessary, read the lesson or sections of the lesson again.

Reflect Suppose you are explaining the lesson to a friend who was absent from class. Make up your own example for each objective.

Ask yourself how the skills in the lesson connect to skills from earlier lessons in the chapter or book. For Lesson 3-4 you might ask, "How is finding factors like using divisibility tests?"

EXERCISES

- **1.** Answer the questions in the table above for Lesson 3-4.
- **2.** Make up an example for each objective in Lesson 3-4. Explain, in words, how to find the solution to each example.

Look ahead to Lesson 3-5.

- **3.** What is the title and what are the main objectives of the lesson?
- **4.** For each objective of Lesson 3-5, write a question that you should be able to answer after you read the lesson.



Simplifying Fractions

What You'll Learn



To write equivalent fractions



To simplify fractions

... And Why

To find a fraction of road signs, as in Example 4

Check Skills You'll Need

For help, go to Lesson 3-4.

Find the GCF of each pair of numbers.

1. 6, 10

2. 3, 7

3. 12, 24

- 4. 15, 25
- **5.** 45, 50

6. 1, 2

7. Reasoning Why does $3 \div 3$ have the same result as $7 \div 7$?

New Vocabulary • equivalent fractions • simplest form

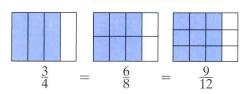
OBJECTIVE



Interactive lesson includes instant self-check, tutorials, and activities.

Writing Equivalent Fractions

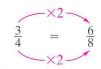
A girls' basketball team won 3 out of 4 games. A boys' soccer team won 6 out of 8 games. Both teams won the same fraction of games. Fractions that name the same amount are equivalent fractions.



Real-World Connection

Careers Coaches encourage their team and share in the joy of winning.

So $\frac{3}{4}$, $\frac{6}{8}$, and $\frac{9}{12}$ are equivalent. You can write equivalent fractions by multiplying or dividing the numerator and the denominator by the same nonzero number.



EXAMPLE

Using Multiples to Write Equivalent Fractions

Use a table of multiples to write three fractions equivalent to $\frac{1}{8}$.

	×2	×3	×4
7	14	21	28
8	16	24	32

 $\leftarrow \begin{array}{l} \text{Multiples in the same column form} \\ \text{fractions equivalent to } \frac{7}{8}. \end{array}$

Three fractions equivalent to $\frac{7}{8}$ are $\frac{14}{16}$, $\frac{21}{24}$, and $\frac{28}{32}$.



- **Check Understanding** 1 a. Use multiples to write two fractions equivalent to $\frac{4}{5}$.
 - **b.** You can write 2 as $\frac{2}{1}$. Write three fractions equivalent to 2.

The fractions you get by dividing the numerator and denominator by a common factor are also equivalent to the original fraction.

EXAMPLE Using Factors to Write Equivalent Fractions

Write three fractions equivalent to $\frac{24}{30}$.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30 List the factors of each number. ← Look for common factors.

$$\frac{24}{30} = \frac{12}{15}$$

$$\frac{24}{30} = \frac{8}{10}$$



Three fractions equivalent to $\frac{24}{30}$ are $\frac{12}{15}$, $\frac{8}{10}$, and $\frac{4}{5}$.



- Check Understanding (2) a. Use common factors to write two fractions equivalent to $\frac{18}{30}$.
 - **b.** Reasoning Are $\frac{2}{4}$ and $\frac{8}{8}$ equivalent? Explain.

OBJECTIVE

Simplifying Fractions

A fraction is written in **simplest form** when the numerator and the denominator have no common factors other than 1. For example, $\frac{1}{3}$ and $\frac{3}{9}$ are equivalent, but only $\frac{1}{3}$ is written in simplest form.

EXAMPLE

Simplifying by Dividing



Another term for simplest form is lowest terms.

Simplify $\frac{12}{24}$.

 $\frac{12 \div 2}{24 \div 2} = \frac{6}{12}$ \leftarrow Divide the numerator and denominator by a common factor.

In simplest form, $\frac{12}{24}$ is $\frac{1}{2}$.



Check Understanding 3 Write $\frac{8}{12}$ in simplest form.

You can also use the GCF to write fractions in simplest form.

EXAMPLE

Using the GCF to Simplify a Fraction



Driving In the United States, there are 48 types of road signs. Of these, 16 are instructional, such as speed limit or stop signs. What fraction of road signs are instructional? Write your answer in simplest form.

 $\frac{16}{48} = \frac{16 \div 16}{48 \div 16} = \frac{1}{3}$ \leftarrow Divide both numerator and denominator by the GCF, 16.

 $\frac{1}{9}$ The fraction of road signs that are instructional is $\frac{1}{3}$.



- **Check Understanding** $\frac{4}{9}$ a. Write $\frac{18}{45}$ in simplest form.
 - **b.** Reasoning Explain why it does not make sense to use the LCM of the numerator and denominator to simplify fractions.



Use multiples to write two fractions equivalent to each fraction.

Example 1 (page 151)

1.
$$\frac{5}{6}$$

2.
$$\frac{3}{8}$$

3.
$$\frac{2}{9}$$

4.
$$\frac{7}{10}$$

6.
$$\frac{3}{5}$$

7.
$$\frac{6}{11}$$

8.
$$\frac{1}{5}$$

Example 2 (page 152) Use common factors to write two fractions equivalent to each fraction.

9.
$$\frac{8}{24}$$

10.
$$\frac{18}{36}$$

11.
$$\frac{27}{81}$$
 15. $\frac{24}{84}$

12.
$$\frac{60}{140}$$

13.
$$\frac{30}{42}$$

14.
$$\frac{45}{90}$$

11.
$$\frac{27}{81}$$
15. $\frac{24}{84}$

16.
$$\frac{36}{80}$$

Examples 3 and 4 (page 152) Write each fraction in simplest form.

17.
$$\frac{24}{32}$$

18.
$$\frac{18}{27}$$
19. $\frac{33}{39}$ 20. $\frac{8}{18}$ 22. $\frac{21}{28}$ 23. $\frac{18}{30}$ 24. $\frac{25}{35}$

19.
$$\frac{33}{39}$$

20.
$$\frac{8}{18}$$

21.
$$\frac{16}{28}$$

22.
$$\frac{21}{28}$$

23.
$$\frac{18}{30}$$

24.
$$\frac{25}{35}$$

- **25. Biology** An adult's body has 206 bones. Of these, 106 are in the feet, ankles, wrists, and hands. What fraction of an adult's bones is in the feet, ankles, wrists, and hands? Write your answer in simplest form.
- B Apply Your Skills

Use the table. Find the number that makes each statement true.

26.
$$\frac{5}{6} = \frac{20}{6}$$

26.
$$\frac{5}{6} = \frac{20}{10}$$
 27. $\frac{25}{30} = \frac{1}{48}$

- **28.** $\frac{30}{36} = \frac{15}{42}$ **29.** $\frac{35}{24} = \frac{35}{42}$
- 30. a. Data File, p. 129 Write the shutter opening times as equivalent fractions that all have the same denominator.
 - **b.** How could you use the fractions you wrote to compare the times?
- **31.** a. Find the thickness of the door at the left to the nearest $\frac{1}{2}$ in.
 - **b.** Find the thickness of the door to the nearest $\frac{1}{4}$ in.
 - c. Reasoning A sliding door has the thickness shown at the left. If you are designing a track for the door to slide in, should you use your answer to part (a) or part (b)? Explain.

Write two equivalent fractions for each model.

32.



33.





- 35. Writing in Math Explan how you can use divisibility rules to tell whether the fractions $\frac{2}{10}$ $\frac{10}{24}$, and $\frac{15}{35}$ are in simplest form.
- **36.** Number Sense Wincomnumber is a factor of all positive numbers?



37. Which square does *not* have the same fraction shaded as the others?

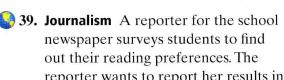






38. Error Analysis A math teacher asks his students to find a fraction equivalent to $\frac{5}{6}$. Shauna's answer is below. Is she correct? Explain.

 $\frac{5}{6} = \frac{5+4}{6+4} = \frac{9}{10}$



reporter wants to report her results in fraction form. For each category, write a fraction in simplest form.

Student Reading Preferences

Category	Number
Novels	18
Biographies	8
Science fiction	4

40. Find an integer that fits the following conditions: It is between 44 and 53. The sum of its digits is a prime number. It has more than 3 factors.





Careers Journalists gather information, analyze data, and write reports.

Find each missing number.

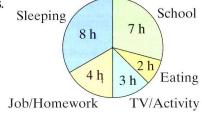
41.
$$\frac{18}{10} = \frac{6}{7}$$

42.
$$\frac{5}{8} = \frac{1}{32}$$

43.
$$\frac{1}{6} = \frac{4}{1}$$

41.
$$\frac{18}{1} = \frac{6}{7}$$
 42. $\frac{5}{8} = \frac{1}{32}$ **43.** $\frac{1}{6} = \frac{4}{1}$ **44.** $\frac{35}{1} = \frac{7}{1}$

45. The chart shows a student's daily activities. Write a fraction in simplest form for the amount of time spent on each activity.



46. Weather The city of Houston, Texas, typically has 75 clear days out of the 365 days in a year. Houston's clear days represent what fraction of a year? Write your answer in simplest form.

Challenge

Algebra Evaluate each expression. Let p = 7, n = -3, and m = 2. Write in simplest form.

47.
$$\frac{p \cdot p}{p \cdot p \cdot p \cdot p}$$

48.
$$\frac{n^2}{n}$$

49.
$$\frac{n^2m^3}{n^3m}$$

- 50. Banking A customer enters a four-digit number to access an ATM. The number formed by the first two digits is divisible by 4 and 7. Three of the factors of the number formed by the last two digits are 4, 5, and 12.
 - **a.** Write three possibilities for the first two digits.
 - **b.** Write one possibility for the last two digits.
 - **c.** The four-digit number is divisible by 3. What is the number?
 - **51. Stretch Your Thinking** In the equations below, \square represents a two-digit number and \triangle represents a three-digit number. What are the numbers?

$$248 - \square = \triangle$$



Test Prep

Reading Comprehension

Read the passage and answer the questions below.



The Global Village

Think of Earth as a global village of only a thousand people. A total of 607 of them are from Asia, 132 are from Africa, 120 are from

Europe, 57 are from South America, 5 are from Australia, and 79 are from North America.

Take It to the NET Online lesson quiz at www.PHSchool.com Web Code: aba-0305

Write each fraction in simplest form.

- 52. the fraction of people from Africa
- 53. the fraction of people from Australia or Asia
- 54. the fraction of people from Europe
- 55. the fraction of people from South America or North America

Mixed Review

Lesson 2-6 Algebra Solve each equation.

56.
$$-5t + 7 = 62$$

57.
$$\frac{s}{2} - 4 = 11$$

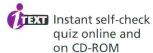
58.
$$\frac{n}{3} + 2 = -4$$

Lesson 1-2 Find each sum or difference.



Checkpoint Quiz 1

Lessons 3-1 through 3-5



Simplify each expression.

1.
$$8^2 + 11$$

2.
$$(-2)^4$$

3.
$$5 + (3^2 - 2)^2$$

Write each number in scientific notation or standard form.

5.
$$2.01 \times 10^4$$

7. The GCF of two numbers is 6. The prime factorization of the first one is $2^2 \cdot 3 \cdot 11$. The second is divisible by 7. What are the numbers?

Simplify each fraction.

8.
$$\frac{18}{36}$$

9.
$$\frac{42}{60}$$

10.
$$\frac{35}{56}$$



Comparing and Ordering Fractions

What You'll Learn



To compare and order fractions

. . . And Why

To compare the sizes of birds around the world, as in Example 2

✓ Check Skills You'll Need



Find the LCM of each pair of numbers.

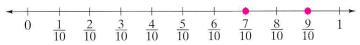
New Vocabulary • least common denominator (LCD)

OBJECTIVE

Interactive lesson includes instant self-check, tutorials, and activities.

Comparing and Ordering Fractions

You can use a number line to compare fractions.





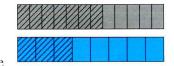
The inequality $\frac{7}{10} < \frac{9}{10}$ has the same meaning as $\frac{9}{10} > \frac{7}{10}$.

If the *denominators* are the same, the numerators tell which is greater. Use the "is greater than" (>) or the "is less than" (<) symbols.

$$\frac{7}{10} < \frac{9}{10}$$

If the numerators of two fractions are the same, the fraction with the lesser denominator has the greater value. For example, $\frac{3}{4} > \frac{3}{5}$.

The fraction models show that $\frac{7}{12} > \frac{3}{8}$. To compare fractions with different denominators, rewrite each with a common denominator. The least common denominator (LCD) of two or more



fractions is the least common multiple (LCM) of their denominators.

EXAMPLE **Comparing Fractions**

Compare $\frac{3}{4}$ and $\frac{9}{10}$.

The denominators are 4 and 10. Their LCM is 20. So, 20 is their LCD.

Check Understanding 1 Compare each pair of fractions. Use <, >, or =.

a.
$$\frac{3}{4} = \frac{5}{6}$$

b.
$$\frac{1}{6} = \frac{2}{9}$$

b.
$$\frac{1}{6} = \frac{2}{9}$$
 c. $\frac{4}{10} = \frac{3}{8}$



Real-World Connection The American phalarope is a water bird. It migrates to

the Arctic in the spring.

You can use the LCD to order more than two fractions.

Real-World Problem Solving **EXAMPLE**

Birds There are many small birds all over the world. The American phalarope is $\frac{2}{3}$ ft long, the African cuckoo is $\frac{3}{4}$ ft long, and the Eurasian skylark is $\frac{7}{12}$ ft long. Which of these birds is the smallest? Which is the largest? Order the birds from least to greatest in length.

Order $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{7}{12}$.

The LCM of 3, 4, and 12 is 12. So 12 is the LCD of the three fractions.

$$\begin{array}{l} \text{Phalarope} \rightarrow \begin{array}{l} \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \\ \text{Cuckoo} \quad \rightarrow \begin{array}{l} \frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \end{array} \end{array} \right\} \leftarrow \text{Use the LCD to write equivalent fractions.}$$

$$\text{Skylark} \quad \rightarrow \begin{array}{l} \frac{7}{12} \end{array}$$

$$\frac{7}{12} < \frac{8}{12} < \frac{9}{12}$$
. So, $\frac{7}{12} < \frac{2}{3} < \frac{3}{4}$. \leftarrow Compare the numerators.

The skylark is the smallest of these three birds. The cuckoo is the largest.



- **a.** $\frac{3}{8}, \frac{1}{2}, \frac{2}{5}$
- **b.** $\frac{6}{9}, \frac{1}{2}, \frac{7}{12}$
- c. $\frac{1}{5}, \frac{2}{6}, \frac{1}{15}$
- **d. Reasoning** To order $\frac{4}{10}$, $\frac{3}{5}$, and $\frac{5}{25}$, it is helpful to write each fraction in simplest form first, before considering the LCD. Explain why.

For more practice, see Extra Practice.



Compare each pair of fractions. Use <, >, or =.

Example 1 (page 156)

1.
$$\frac{5}{12} = \frac{7}{12}$$

2.
$$\frac{5}{6} = \frac{3}{6}$$

3.
$$\frac{1}{3} = \frac{3}{4}$$

4.
$$\frac{5}{6} = \frac{3}{5}$$

5.
$$\frac{3}{8} = \frac{2}{3}$$

6.
$$\frac{6}{7} = \frac{4}{5}$$

7.
$$\frac{2}{3} = \frac{5}{8}$$

1.
$$\frac{5}{12} \blacksquare \frac{7}{12}$$
 2. $\frac{5}{6} \blacksquare \frac{3}{6}$
 3. $\frac{1}{3} \blacksquare \frac{3}{4}$
 4. $\frac{5}{6} \blacksquare \frac{3}{5}$

 5. $\frac{3}{8} \blacksquare \frac{2}{3}$
 6. $\frac{6}{7} \blacksquare \frac{4}{5}$
 7. $\frac{2}{3} \blacksquare \frac{5}{8}$
 8. $\frac{5}{6} \blacksquare \frac{7}{10}$

9.
$$\frac{3}{8} = \frac{3}{5}$$

10.
$$\frac{3}{9} = \frac{1}{3}$$

11.
$$\frac{3}{4} = \frac{3}{10}$$

9.
$$\frac{3}{8} = \frac{3}{5}$$
 10. $\frac{3}{9} = \frac{1}{3}$ **11.** $\frac{3}{4} = \frac{3}{10}$ **12.** $\frac{1}{8} = \frac{3}{16}$

Example 2 (page 157) Order from least to greatest.

13.
$$\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$$

14.
$$\frac{3}{8}, \frac{1}{4}, \frac{2}{3}$$

15.
$$\frac{4}{9}, \frac{2}{3}, \frac{1}{2}$$

16.
$$\frac{1}{3}, \frac{5}{6}, \frac{3}{8}$$

17.
$$\frac{1}{8}, \frac{1}{6}, \frac{1}{9}$$

18.
$$\frac{3}{15}, \frac{3}{10}, \frac{3}{5}$$

19.
$$\frac{5}{8}, \frac{7}{9}, \frac{2}{1}$$

20.
$$\frac{6}{10}, \frac{7}{12}, \frac{5}{8}$$

21.
$$\frac{2}{5}$$
, $\frac{3}{20}$, $\frac{4}{5}$

② 22. Languages In Europe, $\frac{1}{3}$ of the languages spoken are Romance languages, $\frac{2}{15}$ are Germanic, and $\frac{8}{25}$ are Balto-Slavic. Order the language categories from least to greatest.

Apply Your Skills

Compare each pair of fractions. Use <, >, or =.

23.
$$\frac{7}{12} = \frac{5}{9}$$

24.
$$\frac{10}{15}$$
 \blacksquare $\frac{16}{24}$

25.
$$\frac{8}{16} = \frac{15}{32}$$

24.
$$\frac{10}{15} = \frac{16}{24}$$
 25. $\frac{8}{16} = \frac{15}{32}$ **26.** $\frac{22}{26} = \frac{10}{13}$

Use the table at the right.

- 27. Do people remember more of what they say or more of what they do?
- **28.** Do people remember more of what they hear or more of what they say?

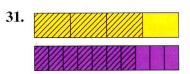
People remember	of
three fourths	what they say.
one tenth	what they hear.
nine tenths	what they do.

Memory Facts

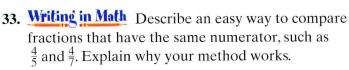
29. Order what people remember from greatest to least.

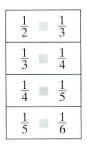
Write two fractions for the models and compare them. Use <, >, or =.





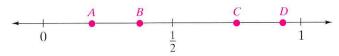
32. Patterns Copy the table. Compare the fractions and fill in your answers. Use <, >, or =.





- **34.** Data File, p. 129 A fast shutter speed allows less light in because the shutter is open for less time.
 - a. Which shutter opening time represents the fastest shutter speed?
 - **b.** A camera is adjusted to a shutter opening time of $\frac{1}{8}$ s. Order the shutter opening times, including $\frac{1}{8}$ s, from least to greatest.

Match each fraction with a point on the number line below.



35.
$$\frac{3}{8}$$

36.
$$\frac{11}{12}$$

37.
$$\frac{3}{4}$$

38.
$$\frac{3}{16}$$

Order from least to greatest.

39.
$$1, \frac{4}{6}, \frac{1}{3}$$

40.
$$\frac{10}{15}, \frac{6}{10}, \frac{1}{3}$$
 41. $2, \frac{5}{2}, \frac{4}{3}$ **42.** $\frac{1}{8}, \frac{3}{12}, \frac{4}{10}$

41.
$$2, \frac{5}{2}, \frac{4}{3}$$

42.
$$\frac{1}{8}$$
, $\frac{3}{12}$, $\frac{4}{10}$

Which fractional part would you prefer? Explain and show your work.

43.
$$\frac{5}{6}$$
 or $\frac{3}{4}$ hours of dental work

44.
$$\frac{1}{6}$$
 or $\frac{1}{12}$ of a year on vacation

45.
$$\frac{2}{9}$$
 or $\frac{4}{15}$ of your paycheck

- **《 46. Carpentry** You want to nail a board that is $\frac{1}{2}$ in. thick onto a wall. You can choose from nails that are $\frac{3}{8}$ in. long and $\frac{3}{4}$ in. long. Which size nail is the better choice? Explain.
 - **47.** Reasoning Your class orders the same number of cheese, vegetable, and meatball pizzas. There are $\frac{5}{8}$ of cheese, $\frac{2}{3}$ of vegetable, and $\frac{3}{4}$ of meatball pizza left over. Which is most popular? Explain.



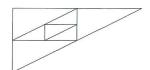
Algebra Compare. Use <, >, or =. The variable n is a positive integer.

48.
$$\frac{1}{n}, \frac{1}{n+1}$$

49.
$$\frac{n}{2}, \frac{n+1}{2}$$

50.
$$\frac{1}{n}, \frac{1}{2n}$$

51. Stretch Your Thinking Draw exactly four triangles to make the figure at the right.





Test Prep

Multiple Choice

52. To make a noodle casserole, Jack uses $\frac{1}{3}$ cup of cream cheese, Myra uses $\frac{2}{5}$ cup, Donna uses $\frac{3}{4}$ cup, and Ferron uses $\frac{1}{2}$ cup. Who uses the most?





53. On a group project, Richard does $\frac{1}{4}$ of the work, Bonita does $\frac{3}{16}$, Gary does $\frac{1}{8}$, and Sierra does $\frac{7}{16}$. Who does the least work?

- I. Sierra
- 54. Which of the following is NOT written in order from least to

greatest?
A.
$$\frac{2}{9}$$
, $\frac{3}{12}$, $\frac{5}{6}$

B.
$$\frac{1}{2}$$
, $\frac{5}{6}$, $\frac{3}{8}$

B.
$$\frac{1}{2}$$
, $\frac{5}{6}$, $\frac{3}{3}$ **C.** $\frac{3}{12}$, $\frac{5}{6}$, $\frac{2}{9}$

D.
$$\frac{2}{9}$$
, $\frac{5}{6}$, $\frac{3}{3}$

Short Response

- 55. A bank offers three types of interest-bearing savings accounts. One account increases by $\frac{1}{2}$ percent annually, another increases by $\frac{3}{5}$ percent, and a third increases by $\frac{7}{12}$ percent.
 - a. Order the fractions from least to greatest.
 - b. In which account would you prefer to invest your money?

Mixed Review

Algebra Find each sum. Lesson 1-7

56.
$$-3 + 7$$

57.
$$5 + (-2)$$

58.
$$-6 + (-8)$$

Lesson 1-3 Find each product or quotient.



Solve a Simpler Problem and Look for a Pattern

What You'll Learn



To solve a problem by combining strategies

... And Why

To solve a multi-step problem, as in Example 1

Check Skills You'll Need

For help, go to Lesson 3-1.

Simplify each expression.

- **1.** 4²
- **3.** 7³
- 5. 4¹

- **2.** 5⁴
- **4.** 3⁵
- 6.8^2

OBJECTIVE



Interactive lesson includes instant self-check, tutorials, and activities.

Solving a Problem by Combining Strategies

When to Use These Strategies If a problem seems to have a large number of steps, you may be able to solve a simpler problem first. Then look for a pattern that will give you a clue to the solution of the original problem.

EXAMPLE

Combining Strategies

When you simplify 3^{50} , what number is the ones place?

Read and Understand You know that 3^{50} is a large number to calculate. You need to find out what number is in the ones place.

Plan and Solve It is not easy to simplify 3^{50} with paper and pencil. You could simplify easier expressions, like 3^2 , 3^3 , and 3^4 , to see what number is in the ones place.

Step 1: Solve a Simpler Problem

Find the values of the first 10 powers of 3. Refer to the table at the right. Notice that the ones digits in the value column repeat in the pattern 3, 9, 7, 1

Step 2: Look for a Pattern

Organize the information to see whether there is a connection between the ones digits and the exponents. Refer to the table below.

Ones Place	Exponent of 3
3	1, 5, 9
9	2, 6, 10
7	3,7
1	4,8

Power	Value
31	3
32	9
3^3	27
3 ⁴	81
3 ⁵	243
3 ⁶	729
3 ⁷	2,187
3 ⁸	6,561
3 ⁹	19,683
3 ¹⁰	59,049

If the exponent is divisible by 4, then the ones digit is 1. Since 48 is divisible by 4, you can start there:

Exponent:

50

Ones digit:

1

3

9

The number in the ones place of 3^{50} is 9.

Look Back and Check Look for a pattern in another row of the exponent table that might also work.

When the exponent of 3 is 2, 6, or 10, the ones digit is 9. The numbers 2, 6, and 10 are divisible by 2, but not by 4. Since 50 is divisible by 2 but not by 4, the ones digit of 3^{50} is 9.

- Check Understanding 1 a. Describe the pattern for the ones digit of any power of 8.
 - **b.** When you simplify 8^{63} , what number is in the ones place?

EXERCISES

For more practice, see Extra Practice.



Solve each problem by solving a simpler problem and looking for a pattern.

Example 1 (page 160)

- **1. a.** What is the pattern for the ones digit of any power of 7? **b.** When you simplify 7^{21} , what number is in the ones place?
- 2. The table shows the values of powers of 2 with even exponents from 10 to 20.
 - **a.** Make a table that lists the exponents of 2 and their corresponding ones digit.
 - **b.** What is the ones digit of 2^{80} ?
- **3.** What is the value of $(-1)^{427}$? Explain your reasoning.
- **4.** When you simplify 10^{347} , what number is in the ones place?

Power	Value
2 ¹⁰	1,024
212	4,096
214	16,384
216	65,536
218	262,144
2 ²⁰	1,048,576



Use any strategy or combination of strategies to solve each problem.

Need Help?

- Reread the problem.
- Identify the key facts and details.
- Tell the problem in your own words.
- Try a different strategy.
- Check your work.

- **5. Sports** There are 64 teams in a state soccer tournament. A team is eliminated if it loses a game. How many games must be played to determine the state soccer champion?
- **6. Cars** A new car comes in five different exterior colors and three different interior colors. How many different color combinations are available?
- 7. Nita, a six-year-old girl, can stretch her legs to take two stairs at one time. How many different ways can she climb six stairs using any combination of one or two stairs?

Strategies

Draw a Diagram
Look for a Pattern
Make a Graph
Make an Organized List
Make a Table
Simulate a Problem
Solve a Simpler Problem
Try, Check, and Revise
Use Logical Reasoning
Work Backward
Write an Equation

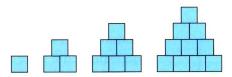
8. Writing in Math To the amazement of his teacher and classmates, ten-year-old Carl Friedrich Gauss found the sum of the first 100 positive integers. His method is shown below for the first 8 positive integers. How would you find the sum of the first 50 positive integers using his method?

പ്രത്യത്തെന്ന് എള് എത്രില് പോട്ടത്തെന്ന് എള്ളുന്ന് നിന്ന് പോട്ടത്തിന്റെ പ്രത്യത്തെന്ന് എള്ളുന്ന് നിന്ന് പോട്ടത്തെന്

- **9. Business** A worker is hired for \$80 a day on the condition that if business is slow, the worker will only receive half pay. At the end of 20 days, the worker receives \$1,320. On how many days was business slow?
- **10.** Lewis has between 3 and 100 toy cars in his collection. If he counts them three, four, or five cars at a time, he always has 2 left over. How many toy cars does Lewis have?



- 11. Health A 125-lb person burns 110 Calories walking at 2 mi/h for an hour. That same person burns 180 Calories walking at 3 mi/h, and 260 Calories walking at 4 mi/h. How many Calories do you think a 125-lb person burns in 1 hour by walking 5 mi/h?
 - 12. a. Patterns How many boxes will be in the next arrangement?b. Draw the next arrangement.



- **C**hallenge
- **13. Number Sense** Find the sum of the first 20 powers of 10, that is, the sum of $10^1 + 10^2 + 10^3 + \dots + 10^{20}$.
- **14. Stretch Your Thinking** You are sitting on chair 33 of a ski lift, which is a loop. Chairs are numbered consecutively, starting with 1. Chair 97 passes you halfway up the ski slope. How many chairs are on the lift?



Test Prep

Multiple Choice

15. Which expression is equivalent to
$$9m + 9n$$
?

A.
$$18(m + n)$$

B.
$$9(m + n)$$

C.
$$9m + n$$

$$D.m + n$$

Online lesson quiz at www.PHSchool.com Web Code: aba-0307

16. Which equation is equivalent to
$$8p + 4.5 = 32$$
?

F.
$$8p + 4.5 = 32 + 4.5$$

G.
$$8p = 36.5$$

H.
$$8p = 27.5$$

1.
$$6p = 32 - 2$$

17. If
$$4t + 13.7 = 29.9$$
, what is the value of t ?

Short Response

18. Medium beverages cost \$1.39 and small beverages cost \$.89. Write an equation to find the total cost in dollars (d) of any number of medium beverages (m) and small beverages (s).

Mixed Review

Lesson 3-4 Find the LCM of each pair of numbers.

Lesson 3-3 Tell whether the first number is divisible by the second. Explain.

Lesson 2-6

Algebra Solve each equation.

27.
$$2m + 3 = 25$$
 28. $\frac{w}{4} - 3 = 3$ **29.** $6m - 7 = 29$

28.
$$\frac{w}{4} - 3 = 3$$

29.
$$6m - 7 = 29$$

30.
$$\frac{1}{3}t + 7 = 27$$
 31. $5h - 2 = 73$ **32.** $\frac{1}{4}x - 3 = 17$

31.
$$5h - 2 = 7$$

32.
$$\frac{1}{4}x - 3 = 17$$

Practice Game

Factor Cards



• 40 index cards numbered from 1 through 40.

How to Play

- Divide the class into two teams.
- Team A chooses a card.
- Team B picks up any of the remaining cards that are factors of Team A's card. For example, if Team A chooses 10, Team B picks up cards 1, 2, and 5.
- Teams switch places until all of the cards are picked up. The team with the highest sum of factors on its cards wins.







Mixed Numbers and Improper Fractions

What You'll Learn



To write a mixed number as an improper fraction



To write an improper fraction as a mixed number

. . . And Why

To plan a pizza party, as in Example 3

Check Skills You'll Need

For help, go to Lesson 3-5.

Write each fraction in simplest form.

- 1. $\frac{12}{20}$
- 3. $\frac{24}{36}$
- 5. $\frac{20}{48}$

- 2. $\frac{15}{18}$

New Vocabulary • improper fraction • mixed number

OBJECTIVE



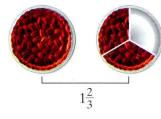
Interactive lesson includes instant self-check, tutorials, and activities.

Writing Mixed Numbers as Improper Fractions

The fractions $\frac{5}{2}$, $\frac{12}{12}$, and $\frac{8}{3}$ are improper fractions. An improper fraction has a numerator that is greater than or equal to its denominator.

The numbers $2\frac{1}{2}$, $1\frac{2}{3}$, and $3\frac{2}{5}$ are mixed numbers. A **mixed number** is the sum of a whole number and a fraction.

The models below show that $1\frac{2}{3} = \frac{5}{3}$.







A number line can also help you understand improper fractions and mixed numbers.



The number line shows that $1\frac{1}{4} = \frac{5}{4}$.

One way to write a mixed number as an improper fraction is to write the mixed number as a sum. Write a fraction that is equivalent to the whole number, and then find the sum of the fractions.

1 EXAMPLE Writing an Improper Fraction

Write $2\frac{3}{4}$ as an improper fraction.

$$\begin{aligned} 2\frac{3}{4} &= 2 + \frac{3}{4} & \leftarrow \text{Write the mixed number as a sum.} \\ &= \frac{8}{4} + \frac{3}{4} & \leftarrow \text{Write 2 as a fraction with the same denominator as } \frac{3}{4}. \\ &= \frac{8+3}{4} = \frac{11}{4} & \leftarrow \text{Add the numerators.} \end{aligned}$$

Check Understanding 1 Write $3\frac{3}{4}$ as an improper fraction.

Here is another method you can use to write improper fractions.

2 EXAMPLE Using Multiplication

Write $4\frac{2}{3}$ as an improper fraction.

Multiply the denominator by the whole number. Add the numerator. $4\frac{2}{3} = 4 + 2\frac{2}{3} = \frac{(3 \times 4) + 2}{3} = \frac{14}{3}$

The denominator stays the same.

Check Understanding 2 Write $2\frac{5}{8}$ as an improper fraction.

OBJECTIVE

Writing Improper Fractions as Mixed Numbers

To write an improper fraction as a mixed number, divide and write the remainder as a fraction of the denominator. Then simplify the fraction.

3 EXAMPLE Writing a Mixed Number World

Party Planning You are planning a pizza party. Each pizza has 8 slices. You estimate you will need 30 slices. How many pizzas should you order?

To find the number of pizzas, write $\frac{30}{8}$ as a mixed number.

$$\begin{array}{c} 3 \\ \text{denominator} \rightarrow 8)\overline{30} \\ \underline{-24} \\ \overline{6} \end{array} \leftarrow \text{remainder} \\ 3\frac{6}{8} = 3\frac{3}{4} \end{array} \leftarrow \text{Write the remainder as a fraction, } \frac{\text{remainder}}{\text{denominator}}. \text{ Simplify.}$$

Since you cannot order $3\frac{3}{4}$ pizzas, you should order 4 pizzas.

- **Check Understanding** 3 a. Reasoning How can you use the GCF before you divide? Explain.
 - **b.** Write $\frac{15}{12}$ as a mixed number in simplest form.



Write each mixed number as an improper fraction.

Examples 1 and 2 (page 165)

1.
$$2\frac{3}{8}$$
2. $5\frac{3}{4}$ 3. $1\frac{1}{12}$ 4. $4\frac{3}{5}$ 5. $1\frac{3}{7}$ 6. $4\frac{5}{8}$ 7. $3\frac{2}{5}$ 8. $2\frac{11}{12}$ 9. $5\frac{2}{3}$ 10. $9\frac{1}{4}$

3.
$$1\frac{1}{1}$$

4.
$$4\frac{3}{5}$$

5.
$$1\frac{3}{7}$$

6.
$$4\frac{5}{8}$$

7.
$$3\frac{2}{5}$$

8.
$$2\frac{11}{12}$$

9.
$$5\frac{2}{3}$$

10.
$$9\frac{1}{4}$$

11.
$$4\frac{5}{6}$$

12.
$$2\frac{4}{9}$$

11.
$$4\frac{5}{6}$$
 12. $2\frac{4}{9}$ **13.** $4\frac{7}{10}$ **14.** $7\frac{3}{5}$

14.
$$7\frac{3}{5}$$

15.
$$6\frac{4}{7}$$

Example 3 (page 165) Write each improper fraction as a mixed number in simplest form.

16.
$$\frac{16}{3}$$

17.
$$\frac{25}{3}$$
 18. $\frac{42}{4}$ **19.** $\frac{31}{12}$ **20.** $\frac{28}{6}$

18.
$$\frac{42}{4}$$

19.
$$\frac{31}{12}$$

20.
$$\frac{28}{6}$$

21.
$$\frac{49}{6}$$

21.
$$\frac{49}{6}$$
 22. $\frac{40}{6}$ **23.** $\frac{45}{10}$ **24.** $\frac{48}{11}$ **25.** $\frac{15}{8}$

23.
$$\frac{45}{10}$$

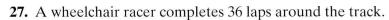
24.
$$\frac{48}{11}$$

25.
$$\frac{15}{8}$$

26. Baking Challah is a bread made by braiding 6 strands of dough together. What is the greatest number of loaves of challah that can be made with 40 strands of dough?







28. A jogger completes 40 laps around the track.



Real-World Connection Wheelchair racing became an Olympic sport in 1960.

Write each improper fraction as a whole or mixed number in simplest form.

29.
$$\frac{9}{2}$$
 30. $\frac{42}{7}$ **31.** $\frac{22}{5}$ **32.** $\frac{17}{6}$ **33.** $\frac{27}{4}$ **34.** $\frac{19}{12}$ **35.** $\frac{18}{4}$ **36.** $\frac{21}{9}$ **37.** $\frac{32}{8}$ **38.** $\frac{29}{4}$

31.
$$\frac{22}{5}$$

32.
$$\frac{17}{6}$$

33.
$$\frac{27}{4}$$

34.
$$\frac{19}{12}$$

35.
$$\frac{18}{4}$$

36.
$$\frac{21}{9}$$

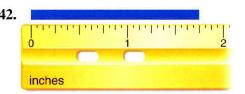
37.
$$\frac{32}{8}$$

38.
$$\frac{29}{4}$$

- **39.** Number Sense Write a fraction with the greatest possible value using each of the digits 2, 5, and 9 exactly once. Then write the fraction as a mixed number.
- **3 40. Clothing** A tailor designs a skirt that is $25\frac{1}{4}$ in. long. What is the length in eighths of an inch? Write your answer as an improper fraction.

Write each length as a mixed number and as an improper fraction.





Write each mixed number as an improper fraction.

43.
$$8\frac{2}{3}$$

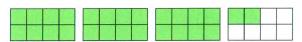
44.
$$5\frac{8}{9}$$

44.
$$5\frac{8}{9}$$
 45. $6\frac{1}{7}$ **46.** $7\frac{3}{5}$ **47.** $12\frac{4}{7}$

46.
$$7\frac{3}{5}$$

47.
$$12\frac{4}{7}$$

- **48.** Writing in Math Which is longer, $\frac{9}{4}$ miles or $1\frac{1}{2}$ miles? Explain.
- **49.** Modeling Write a mixed number and an improper fraction for the model below.





Algebra For Exercises 50–52, evaluate each expression for a = 6, b = 3, c = 2, and d = 5. Write your answers in simplest form.

50.
$$\frac{b}{a^2}$$

51.
$$\frac{a^2 + b}{c}$$

52.
$$\frac{a+c}{b+d}$$

53. Stretch Your Thinking Five friends are seated in a row at a concert. Shayla is sitting next to Suki. Sam is in the middle of the row and sitting next to Suki and Sari. Shar is sitting on the right end of the row. Who is sitting on the left end of the row?



Test Prep

Multiple Choice



54. A piece of fabric measures between 3 and $3\frac{1}{4}$ ft. Which length could it be?

A.
$$\frac{72}{25}$$
 ft

B.
$$\frac{25}{8}$$
 ft

B.
$$\frac{25}{8}$$
 ft **C.** $\frac{18}{5}$ ft

D.
$$\frac{26}{7}$$
 ft

55. You want to buy sports drinks for the players on your basketball team. If each drink costs \$1.69, how many drinks can you buy with \$22?

- I. 15
- **56.** Choose the mixed number that is equivalent to $\frac{14}{6}$. **A.** $2\frac{1}{2}$ **B.** $\frac{7}{2}$ **C.** $1\frac{2}{3}$

A.
$$2\frac{1}{3}$$

B.
$$\frac{7}{3}$$

C.
$$1\frac{2}{3}$$

D.
$$\frac{6}{14}$$

Extended Response

- 57. A carpenter spends a total of 345 minutes on a project. The carpenter is being paid \$29.50 per hour.
 - a. Use mixed numbers to write the time in hours. Show your work.
 - b. Estimate the amount the carpenter is paid. Explain how you found your estimate.

Mixed Review

Algebra Evaluate each expression for a = 10, b = -2, and c = 3. Lesson 2-1

60.
$$abc - 1$$

61.
$$b + 3c - b$$

Find each difference. Lesson 1-7



Fractions and Decimals

What You'll Learn



To relate fractions and decimals

. And Why

To compare enrichment class preferences, as in Example 4

Check Skills You'll Need

For help, go to Lesson 3-5.

Write each fraction in simplest form.

1. $\frac{8}{10}$

4. $\frac{70}{100}$

5. $\frac{24}{36}$

New Vocabulary • terminating decimal • repeating decimal

OBJECTIVE



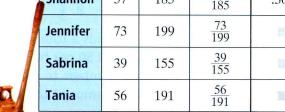
্বিত্রের Interactive lesson includes instant self-check, tutorials, and activities.

Relating Fractions and Decimals

Investigation: Ordering Fractions

The ball speeds toward you. Pow! You snap the bat for a hit. It doesn't happen every time. In fact, most players get a hit less than $\frac{1}{3}$ of the time.

Player	Hits	Times at Bat	Fraction of Hits	Batting Average
Shannon	57	185	57 185	.308
Jennifer	73	199	73 199	M
Sabrina	39	155	39 155	III.
Tania	56	191	<u>56</u> 191	11.



- 1. Calculator Copy the table. For each player's fraction of hits, divide the numerator by the denominator. Round to the nearest thousandth. Fill in your answers in the Batting Average column for each player.
- 2. Which player had the highest batting average? List the players in order, from highest to lowest batting averages.
- **3.** Reasoning Which is easier to compare, the fraction of hits for each player or the decimal batting average for each player? Explain.

You can write a fraction as a decimal by dividing the numerator by the denominator. A decimal that stops, or terminates, is a terminating decimal.

EXAMPLE

Writing a Terminating Decimal



Chemistry The concentration of nitrogen in a chemical sample is $\frac{5}{8}$. Write the fraction of nitrogen as a decimal.

Method 1 Paper and Pencil

Method 2 Calculator

$$\frac{5}{8} \text{ or } 5 \div 8 = 8)5.000$$

$$\frac{-48}{20}$$

$$\frac{-16}{40}$$

$$0 \leftarrow \text{The remainder is 0.}$$

So, $\frac{5}{8} = 0.625$. The nitrogen concentration is 0.625.

Check Understanding (1) a. Write $\frac{4}{5}$ as a decimal.

b. Reasoning Is a remainder of 0 the same as no remainder? Explain.

Reading Math

The symbol for a repeating decimal is a bar over the repeated digit(s), such as $0.\overline{17}$.

If the same block of digits in a decimal repeats without end, the decimal is a repeating decimal. The repeating block can be one or more digits.

EXAMPLE Writing a Repeating Decimal

Write $\frac{3}{11}$ as a decimal.

Calculator Hint

Most calculators display as many digits as possible of a repeating decimal and round off the final digit. For example, 0.27 might be shown as 0.272727273.

Method 1 Paper and Pencil

$$\frac{3}{11} \text{ or } 3 \div 11 = 11)3.00000$$
The digits "27"
repeat.
$$\frac{-22}{80}$$

$$\frac{-77}{30}$$

$$\frac{-22}{80}$$

$$\frac{-77}{30}$$

$$\frac{-22}{80}$$

$$\frac{-77}{30}$$

 $30 \leftarrow$ There will always be a remainder.

So,
$$\frac{3}{11} = 0.\overline{27}$$
.



- Check Understanding 2 Write each fraction as a decimal.
- **b.** $\frac{1}{3}$

- e. Reasoning Is 3.0300300030003... a repeating decimal? Explain.

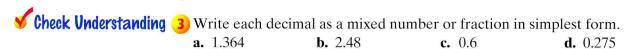
You can write a terminating decimal as a fraction or mixed number by writing the portion of digits to the right of the decimal point as a fraction.

ಗೆ, ಹಾಕೊಂಗಳುಗಿತ್ತದೆ. ಒಂದು ಸರ್ಕಾರಿಗಳ ಗೈ ಹಾಕೊಂಗಳುಗಿತ್ತದೆ ಒಂದು ಕಾಳುವುದುಗಳ ಮತ್ತೂಗಳುಗಿತ್ತದೆ. ಒಂದು ಸರಕ್ಕೆ ಕ್ರೂಗಿ, ಹಾಕ

3 EXAMPLE Writing a Decimal as a Fraction

Write 1.325 as a mixed number with a fraction in simplest form.

Since
$$0.325 = \frac{325}{1,000}$$
, $1.325 = 1\frac{325}{1,000}$.



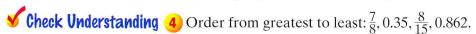
To compare fractions and decimals, write the decimals as fractions or the fractions as decimals. You can decide which is easier for different numbers.

4 EXAMPLE Ordering Fractions and Decimals

Surveys When students are asked which enrichment class they prefer, 0.25 choose sign language, $\frac{10}{48}$ choose starting a business, $\frac{5}{12}$ choose robotics, and 0.125 choose study time. List their choices in order of preference.

$$\begin{array}{l} business \rightarrow \ \frac{10}{48} = \frac{5}{24} = 0.208\overline{3} \\ robotics \rightarrow \ \frac{5}{12} = 0.41\overline{6} \end{array} \right\} \quad \leftarrow \begin{array}{l} \text{Use a calculator to change} \\ \text{the fractions to decimals.} \end{array}$$

Since $0.41\overline{6} > 0.25 > 0.208\overline{3} > 0.125$, students' preferences are robotics, sign language, business, and study time.



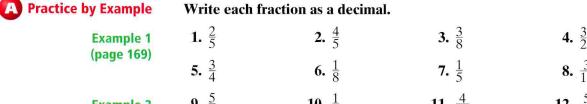
EXERCISES

Reading Math

You read the decimal 1.325 as "one and three hundred twenty-five

thousandths."

For more practice, see Extra Practice.



Example 2 (page 169) 9. $\frac{5}{6}$ 10. $\frac{1}{6}$ 11. $\frac{4}{15}$ 12. $\frac{5}{12}$ 13. $\frac{1}{9}$ 14. $\frac{6}{15}$ 15. $\frac{7}{9}$ 16. $\frac{9}{11}$

Example 3 (page 170) Write each decimal as a mixed number or fraction in simplest form.

- **17.** 0.125
- **18.** 0.66
- 19. 2.5
- **20.** 3.75

- **21.** 0.32
- **22.** 0.19
- **23.** 0.8
- **24.** 0.965

Example 4 (page 170)

Order from greatest to least.

25. $\frac{9}{22}$, 0.83, $\frac{7}{8}$, 0.4

26. 3.84, 3.789, 3.01, $3\frac{41}{50}$

27. $\frac{2}{3}$, 0.67, $\frac{5}{9}$, 0.58, $\frac{7}{12}$

- **28.** $0.1\overline{2}, 0.1225, \frac{3}{25}, \frac{7}{125}$
- **29.** Biology DNA content is measured in picograms (pg). A sea star has $\frac{17}{20}$ pg of DNA, a scallop has $\frac{19}{25}$ pg, a red water mite has 0.19 pg, and a mosquito has 0.024 pg. Order their DNA contents from greatest to least.



Order from least to greatest.

- **30.** $3.\overline{6}, \frac{16}{5}, 3\frac{5}{6}$
- **31.** $1.\overline{01}$, $1\frac{1}{100}$, 1.0101 **32.** $\frac{3}{2}$, $1\frac{2}{5}$, $1.\overline{3}$
- 33. Science An experiment with plant seeds resulted in the data below.

Seed Type	Α	В	С	D	E	F	G	Н	
Number Sprouted	15	5	22	17	18	21	14	18	8
Number Planted	48	20	44	35	52	63	55	35	15

- **a.** For each seed type, write the fraction of $\frac{\text{number sprouted}}{\text{number planted}}$.
- b. Calculator Write each fraction as a decimal. Round to the nearest hundredth.
 - c. Place the seed types into 3 groups: those that sprout about $\frac{1}{2}$ of the time, about $\frac{1}{3}$ of the time, and about $\frac{1}{4}$ of the time.
 - **d.** Which type(s) of seeds would you prefer to plant? Explain.



- 34. Geography About 12,500 icebergs break away from Greenland each year. Of these, about 375 float into the Atlantic Ocean.
 - a. What fraction of the icebergs float into the Atlantic Ocean?
 - **b.** Write your answer for part (a) as a decimal.
 - **c.** What fraction of the icebergs does *not* float into the Atlantic Ocean?



Greenland is the world's largest island. Its area is more than 800,000 square miles.

For Exercises 35–37, use the table at the right.

- 35. For each state, write a fraction that shows the number of people under age 18 total population
- **36. Estimation** For most of the states, would $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ best describe the fraction of the population that is under age 18?
- **37.** Calculator Order the states from least to greatest fraction under age 18.

Population (thousands)

State	Total	Under Age 18
N.Y.	18,976	4,690
Texas	20,852	5,887
Calif.	33,872	9,250
Fla.	15,982	3,646
Ohio	11,353	2,888

Source: U.S. Census Bureau. Go to www.PHSchool.com for a data update. Web Code: abg-2041

38. a. Data File, p. 129 Write each shutter opening time as a decimal. Round to the nearest thousandth.

조리, #15.4 전화보다 보고 보다 바다가 하면 하, #15.4 전화보이다 보고 보다 #15.4 전환 40 전환 40 전환 40 전환 42 전환 42 전환 42 전환 42 전환 42 전환 42 전환

- **b.** Is it easier to compare the times as decimals or fractions? Explain.
- 39. Writing in Math Describe some everyday situations in which you need to change fractions to decimals.

Challenge

(Algebra) Compare. Use <, >, or =. The variable n is a value greater than 1.

40.
$$\frac{1}{n} = \frac{n}{n}$$

41.
$$1 = \frac{n}{1}$$

42.
$$n = \frac{1}{n}$$

41.
$$1 = \frac{n}{1}$$
 42. $n = \frac{1}{n}$ **43.** $\frac{n}{n^2} = \frac{1}{n}$

- **44.** Reasoning Divide 50 by these numbers: 100, 10, 1, 0.1, 0.01, 0.001. As the divisor gets closer to 0, what happens to the quotient? Explain.
- **45. Stretch Your Thinking** I am an integer 28 units from my opposite value on the number line. What two integers could I be?



est Prep

Multiple Choice

- **46.** The top batting averages on a girls' softball team are Jessi $\frac{47}{164}$, Chrissy $\frac{35}{137}$, Carmen $\frac{23}{85}$, and Heather $\frac{32}{93}$. Who has the highest average?
 - A. Jessi
- B. Chrissy
- C. Carmen
- D. Heather
- **47.** Order from least to greatest: $\frac{5}{9}$, $\frac{1}{5}$, 0.569, 0.21.

F.
$$\frac{1}{5}$$
, 0.569, $\frac{5}{9}$, 0.21

G. 0.21,
$$\frac{5}{9}$$
, $\frac{1}{5}$, 0.569

H.
$$\frac{5}{9}$$
, 0.21, $\frac{1}{5}$, 0.569

I.
$$\frac{1}{5}$$
, 0.21, $\frac{5}{9}$, 0.569

Short Response



- 48. Player C scores a free throw 51 times out of 99.
 - a. Write player C's free throw rate as a fraction in simplest form.
 - **b.** Write player C's free throw rate as a decimal.

Basketball Players' Statistics

Player	Free Throws	Attempts			
Α	52	96			
В	75	120			
С	51	99			

Mixed Review

Lesson 3-8 Write each mixed number as an improper fraction.

49.
$$1\frac{2}{3}$$

50.
$$2\frac{4}{5}$$

51.
$$6\frac{4}{9}$$

52.
$$3\frac{1}{12}$$

53.
$$4\frac{2}{7}$$

54.
$$1\frac{9}{9}$$

Lesson 1-3 Find each quotient.



Rational Numbers

What You'll Learn



To compare and order rational numbers

... And Why

To order rational numbers from least to greatest, as in Example 3

Check Skills You'll Need

Compare. Use <, >, or =.

1.
$$\frac{8}{10} = \frac{3}{4}$$

2.
$$\frac{10}{50}$$

4.
$$\frac{12}{48} = 0.25$$
 5. $0.45 = \frac{2}{5}$

5. 0.45 =
$$\frac{2}{3}$$

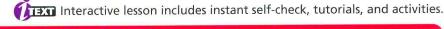
3.
$$1.3 \blacksquare 1\frac{1}{3}$$

6. 0.375
$$\equiv \frac{49}{50}$$

For help, go to Lesson 3-9.

New Vocabulary • rational number

OBJECTIVE



Comparing and Ordering Rational Numbers



Need Help?

Integers are the set of whole numbers, their opposites, and zero.

A rational number is a number that can be written as a quotient of two integers, where the divisor is not 0. Examples are $\frac{2}{5}$, $0.4\overline{6}$, -6, and $3\frac{1}{2}$.

Because you can write any integer as a quotient with a denominator of 1, all integers are rational numbers. For example, you can write 5 as $\frac{5}{1}$.

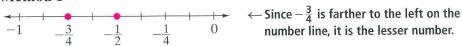
You can write a negative rational number in three ways: $-\frac{7}{9} = \frac{-7}{9} = \frac{7}{-9}$

EXAMPLE

Comparing Negative Rational Numbers

Compare $-\frac{1}{2}$ and $-\frac{3}{4}$.

Method 1



So,
$$-\frac{3}{4} < -\frac{1}{2}$$
.

Method 2

$$-\frac{1}{2} = \frac{-1}{2}$$

 $-\frac{1}{2} = \frac{-1}{2}$ \leftarrow Rewrite $-\frac{1}{2}$ with a -1 in the numerator.

$$=\frac{-1\times2}{2\times2}$$

← The LCD is 4. Write an equivalent fraction.

$$=\frac{-2}{4}=-\frac{2}{4}$$

 $=\frac{-2}{4}=-\frac{2}{4}$ \leftarrow The fraction $-\frac{2}{4}$ is equivalent to $\frac{-2}{4}$.

Since
$$-\frac{3}{4} < -\frac{2}{4}$$
, then $-\frac{3}{4} < -\frac{1}{2}$.



a.
$$-\frac{2}{3} - \frac{1}{6}$$

b.
$$-\frac{1}{4} - \frac{1}{8}$$

a.
$$-\frac{2}{3} - \frac{1}{6}$$
 b. $-\frac{1}{4} - \frac{1}{8}$ **c.** $-\frac{1}{5} - \frac{3}{10}$



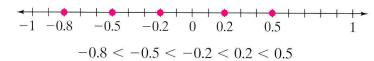
Reading Math

Values are greater as you move to the right on a number line.

Terminating and repeating decimals are rational numbers.

You can use a number line to compare decimals.

사용도 성과 사용 그는 이번 전 내 경에 다른 사용도 사용되어 있다. 아무 전에 내려왔다. 사용도 사용도 아무리 아무리 사용을 하게 다른 사용도 생각하다고 있다. 아무리를 하였다. 남



You can also compare decimals using the signs of the numbers and the digits in which they differ.

EXAMPLE

Comparing Decimals

a. Compare 4.4 and 4.7.

4.4 < 4.7

← Both numbers are positive. Compare the digits.

b. Compare -4.4 and 4.7.

-4.4 < 4.7

 \leftarrow Any negative number is less than a positive number.

c. Compare -4.4 and -4.7.

-4.4 > -4.7

Check Understanding 2 Compare. Use <, >, or =.

a. $-3.5 \equiv 3.8$

b. -4.2 - 4.9 **c.** -8.6 - 8.42

d. Number Sense In Example 2 (c), what do you notice about the tenths place of the greater number, -4.4, when you compare it to the tenths place of the lesser number, -4.7? Explain.

When you compare and order decimals and fractions, it is often helpful to write the fractions as decimals.

Ordering Rational Numbers

Order these numbers from least to greatest: $\frac{1}{4}$, -0.2, $-\frac{2}{9}$, 1.1

$$\frac{1}{4} = 1 \div 4 = 0.25$$

← Write as a decimal.

$$-0.\overline{2} < -0.2 < 0.25 < 1.1$$

← Compare the decimals.

From least to greatest, the numbers are $-\frac{2}{9}$, -0.2, $\frac{1}{4}$, and 1.1.

Check Understanding 3 Order from least to greatest.

a.
$$\frac{2}{3}$$
, -0.1 , $-\frac{5}{8}$, 2.2

b.
$$0.625, \frac{1}{8}, \frac{1}{32}, 0.025$$

c. Reasoning Explain why rounding $-\frac{2}{9}$ to the nearest tenth would not help you compare and order the decimals in Example 3.



Compare. Use <, >, or =.

Example 1 (page 173)

1.
$$-\frac{1}{7} = -\frac{6}{7}$$

1.
$$-\frac{1}{7} \blacksquare -\frac{6}{7}$$
 2. $-\frac{3}{4} \blacksquare -\frac{3}{8}$

3.
$$-\frac{1}{2} - \frac{2}{10}$$

4.
$$-\frac{3}{4}$$

4.
$$-\frac{3}{4} - 1$$
 5. $-\frac{1}{2} - \frac{5}{6}$

6.
$$-\frac{4}{5} - \frac{1}{3}$$

Example 2 (page 174) Compare. Use <, >, or =.

11.
$$-1.22 \blacksquare -6.5$$
 12. $-10.2 \blacksquare -10.23$

Example 3 (page 174) Order from least to greatest.

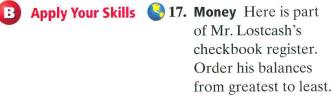
13.
$$\frac{3}{2}$$
, 0.25, $-\frac{3}{4}$, -1.0

14.
$$\frac{7}{3}$$
, 2.4, $-\frac{6}{25}$, -1.34

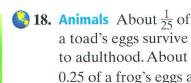
15.
$$\frac{6}{11}$$
, -1.5 , 0.545 , $\frac{1}{2}$

16.
$$-0.8\overline{3}, -\frac{14}{15}, \frac{1}{12}, -0.953$$





Description	Debits (-)	Credits (+)	Balance
Paycheck		122.18	122.18
Sneakers	95.00		27.18
Two outfits	68.09	Wat Raction	-40.91
Paycheck		122.18	81.27
Insufficient			
funds fee	25.00		56.27
Three CDs	59.97		-3.70



0.25 of a frog's eggs and $\frac{1}{5}$ of a green turtle's eggs survive to adulthood. Which animal's eggs have the highest survival rate?

19. Writing in Math Would you compare $-\frac{5}{8}$ and $-\frac{3}{4}$ by finding common denominators or by writing decimal equivalents? Explain your choice.

Compare. Use <, >, or =.

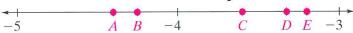
20.
$$-5.8 \equiv -5\frac{9}{10}$$

21.
$$-6\frac{11}{50} \blacksquare -6.21$$

20.
$$-5.8 - 5\frac{9}{10}$$
 21. $-6\frac{11}{50} - 6.21$ **22.** $-10.42 - 10.4\overline{2}$

Real-World Connection Most frogs lay their eggs in

Number Sense Match each number with a point on the number line.



23.
$$-3.6$$
 24. -4.25 **25.** $-3\frac{1}{3}$ **26.** -3.2 **27.** $-4\frac{2}{5}$

25.
$$-3\frac{1}{3}$$

27.
$$-4\frac{2}{5}$$



fresh water because the eggs can develop only in

moist conditions.

- **28.** Algebra Evaluate $\frac{m-n}{-12}$, for m=-3 and n=6.
- 29. Stretch Your Thinking A tour bus travels 400 mi on the first day of a trip. Each day after that the bus travels half the distance it went the day before. The total trip is 775 mi. How many days does the trip take?



Multiple Choice

- 30. The table shows the melting points of chemical solids. Which has the highest?
 - A. Krypton
- B. Argon

ankan bibali, na niberkatikwa 1. ankan kakali na niberkatka 1. ankan bibali na polembakitza 1. ankan kakali na

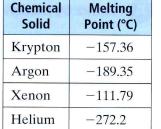
- C. Xenon
- D. Helium
- 31. Which is both an integer and a rational number?

F.
$$-\frac{3}{4}$$

G. 0

H. 0.1

I. $3\frac{3}{5}$



Melting Points

Source: Encyclopædia Britannica



32. Which fraction is NOT equivalent to $-\frac{3}{8}$?

A.
$$\frac{-3}{8}$$

B. $-\frac{9}{24}$

D. $\frac{-6}{16}$

33. Order from least to greatest: $\frac{1}{20}$, 0.056, -0.2, $-\frac{5}{9}$

F.
$$\frac{1}{20}$$
, 0.056, $-\frac{5}{9}$, -0.2

G.
$$-0.2$$
, $-\frac{5}{9}$, $\frac{1}{20}$, 0.056

H.
$$-\frac{5}{9}$$
, -0.2 , $\frac{1}{20}$, 0.056

1.
$$-\frac{5}{9}$$
, -0.2, 0.056, $\frac{1}{20}$

Mixed Review

Lesson 3-9 Order from least to greatest.

34.
$$0.21, \frac{1}{5}, 0.212, \frac{11}{50}$$

35.
$$\frac{21}{25}$$
, $0.9\overline{3}$, $\frac{2}{5}$, 1.99

Write each fraction in simplest form. Lesson 3-5

36.
$$\frac{36}{38}$$

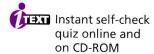
37.
$$\frac{14}{28}$$

38.
$$\frac{12}{56}$$

39.
$$\frac{18}{48}$$



Checkpoint Quiz 2 Lessons 3-6 through 3-10



Compare. Use <, >, or =.

1.
$$\frac{1}{8} = \frac{2}{100}$$

2.
$$\frac{5}{12} = \frac{7}{9}$$

3.
$$\frac{12}{20} = \frac{3}{5}$$

1.
$$\frac{1}{8} = \frac{2}{100}$$
 2. $\frac{5}{12} = \frac{7}{9}$ **3.** $\frac{12}{20} = \frac{3}{5}$ **4.** $2\frac{1}{7} = 0.146$

Write each improper fraction as a mixed number and each mixed number as an improper fraction.

5.
$$\frac{29}{6}$$

6.
$$4\frac{1}{9}$$

7.
$$\frac{82}{5}$$

8.
$$2\frac{5}{6}$$

- 9. Twins occur once in every 89 births. Identical twins occur 4 times in every 1,000 births. Triplets occur once in every 7,900 births. Write each birth frequency as a decimal. Round to the nearest ten-thousandth.
- **10.** Order from least to greatest: $\frac{7}{8}$, $-\frac{4}{9}$, -0.45, $\frac{9}{5}$, 1.89



Writing Extended Responses

An extended response question is usually worth a maximum of 4 points and has multiple parts. To get full credit, you need to answer each part and show all your work or justify your reasoning.

EXAMPLE

Without performing the division, test whether 15,534 is divisible by 3, 4, 9, and 12. Justify each response.

Below are four responses and the amount of credit each received.

4 points

Divisible by 3?

1+5+5+3+4=18

Yes, since 18 is divisible by 3.

Divisible by 4? $34 \div 4 = 8.5$

No, since 34 is not divisible by 4.

Divisible by 9? $18 \div 9 = 2$

Yes, since 18 is divisible by 9.

Divisible by 12?

No, since 15,534 is not divisible by 4.

The 4-point response shows the correct answers and justifies each one.

2 points

Divisible by 3?

1+5+5+3+4=18

Yes, since 18 is divisible by 3.

Divisible by 4? $34 \div 4 = 8$

Yes, since 34 is divisible by 4.

Divisible by 9? $18 \div 2 = 9$

Divisible by 12?

Yes, since 15,534 is divisible

by 3 and 4.

The 2-point response has a computational error and is missing an answer.

3 points

Divisible by 3?

1+5+5+3+4=17

No, since 17 is not divisible by 3.

Divisible by 4? $34 \div 4 = 8.5$

No, since 34 is not divisible by 4.

Divisible by $9?17 \div 9 = 1.8$

No, since 17 is not divisible by 9.

Divisible by 12?

No, since 15,534 is not divisible by 4.

The 3-point response has a computational error, but the student completed both parts.

1 point

Yes, 15,534 is divisible by 3.

No, 15,534 is not divisible by 4.

Yes, 15,534 is divisible by 9.

No, 15,534 is not divisible by 12.

The 1-point response shows correct answers but with no work or justification.

A 0-point response has incorrect answers and no work shown.



Chapter Review

Vocabulary

composite number (p. 146) divisible (p. 141) equivalent fractions (p. 151) exponent (p. 131) factor (p. 146) greatest common factor (GCF) (p. 147) improper fraction (p. 164) least common denominator (LCD) (p. 156) least common multiple (LCM) (p. 145) mixed number (p. 164) multiple (p. 145) power (p. 132) prime factorization (p. 146)

prime number (p. 146) rational number (p. 173) repeating decimal (p. 169) scientific notation (p. 137) simplest form (p. 152) terminating decimal (p. 168)



Reading Math: Understanding Vocabulary

Choose the correct term to complete each sentence.

- 1. In the expression 6^3 , the number 3 represents the (power, exponent).
- 2. A (composite, prime) number has exactly two factors, 1 and itself.
- **3.** (Multiples, Factors) of 28 include 1, 2, 4, 7, 14, and 28.
- **4.** At a class party, you and two friends eat $\frac{6}{8}$ of a pizza. This can also be written as the (improper fraction, equivalent fraction) $\frac{3}{4}$.
- **5.** The (GCF, LCM) of 24 and 36 is 12.

Online vocabulary quiz at www.PHSchool.com Web Code: abj-0351

Take It to the **NET**

Skills and Concepts

3-1 and 3-2 Objectives

- ▼ To write numbers with exponents
- ▼ To simplify expressions with exponents using the Order of Operations
- ▼ To write numbers in scientific notation
- To write numbers in standard form

An **exponent** tells you how many times a number, or base, is used as a factor. A number expressed with an exponent is a **power.** A number in **scientific notation** is written as a product of a factor greater than or equal to 1 but less than 10, and another factor that is a power of 10.

Simplify.

6.
$$-2^4$$

7.
$$(-4)^3$$

8.
$$5^2 + 10^2$$

9.
$$4(5^2 - 10)$$

Write in scientific notation or in standard form.

11.
$$9.06 \times 10^5$$

13.
$$6.015 \times 10^8$$

3-3 Objective

To use divisibility tests

One whole number is **divisible** by a second whole number if the remainder is 0 when you divide the first number by the second.

Test each number for divisibility by 2, 3, 4, 5, 8, 9, and 10.

19. Test 840 for divisibility by 2, 3, 5, 9, and 10.

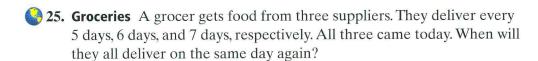
3-4 Objectives

- To find multiples and factors
- To use prime factorization

A **multiple** is the product of a number and any nonzero whole number. A factor is a whole number that divides another whole number with a remainder of 0. A whole number greater than 1 is composite if it has more than two factors and **prime** if it has exactly two factors, 1 and itself.

Write the prime factorization. Use exponents where possible.

- 20, 84
- **21.** 78
- 22. 90
- **23.** 92
- **24.** 125



3-5 and 3-6 Objectives

- To write equivalent fractions
- To simplify fractions
- To compare and order fractions

A fraction is in simplest form when the numerator and denominator have no common factors other than 1. To compare and order fractions, you can use the least common denominator (LCD), which is the least common multiple of their denominators.

Order from least to greatest.

26.
$$\frac{1}{4}$$
, $\frac{1}{3}$, $\frac{1}{6}$

27.
$$\frac{1}{4}$$
, $\frac{2}{5}$, $\frac{3}{8}$

28.
$$\frac{3}{8}$$
, $\frac{5}{6}$, $\frac{1}{2}$

28.
$$\frac{3}{8}$$
, $\frac{5}{6}$, $\frac{1}{2}$ **29.** $\frac{5}{9}$, $\frac{2}{3}$, $\frac{7}{12}$

3-7 and 3-8 Objectives

- To solve a problem by combining strategies
- To write a mixed number as an improper fraction
- To write an improper fraction as a mixed number

When a problem has a large number of steps, you may be able to look for a pattern and solve a simpler problem. An improper fraction has a numerator that is greater than or equal to its denominator. A **mixed number** is the sum of a whole number and a fraction.

30. Traffic Two signs are on a street. One blinks every 10 s and the other every 6 s. How many times per minute do they blink together?

Write each improper fraction as a whole or mixed number in simplest form.

31.
$$\frac{42}{7}$$

32.
$$\frac{27}{12}$$

33.
$$\frac{20}{3}$$

32.
$$\frac{27}{12}$$
 33. $\frac{20}{3}$ **34.** $\frac{125}{5}$

35.
$$\frac{84}{12}$$

3-9 and 3-10 Objectives

- To relate fractions and decimals
- To compare and order rational numbers

To write a fraction as a decimal, you can divide the numerator by the denominator. When the division ends with a remainder of 0, the quotient is a terminating decimal. When the same block of digits in a decimal repeats without end, the quotient is a repeating decimal. A rational number can be written as a quotient of two integers where the denominator is not zero.

Write each fraction as a decimal.

36.
$$\frac{1}{3}$$

37.
$$\frac{5}{9}$$

39.
$$\frac{16}{20}$$

38.
$$\frac{5}{2}$$
 39. $\frac{16}{20}$ **40.** $\frac{4}{50}$

Order from least to greatest.

41.
$$\frac{3}{4}$$
, $0.\overline{3}$, $-\frac{7}{8}$

42. 2.7,
$$-0.3$$
, $-\frac{4}{11}$ **43.** $-\frac{5}{6}$, 2.2, -0.5

43.
$$-\frac{5}{6}$$
, 2.2, -0.5



Chapter Test



Find the value of each expression.

1.
$$(3^2 - 4) \div 5$$
 2. $5^2 - 7^2$

2.
$$5^2 - 7^2$$

3.
$$(6-9)^3$$

4.
$$54 \div 3^2$$

Write using scientific notation.

Tell whether the first number is divisible by the second number.

List 3 factors and 3 multiples of each number.

Find the GCF of each pair of numbers.

18. Reasoning Tell whether each is true or false.

- **a.** Two is a composite number.
- **b.** Any factor of a whole number is greater than any multiple of a whole number.
- **c.** A number is divisible by 3 if its last digit is divisible by 3.
- **d.** One is neither composite nor prime.
- **19.** Use a factor tree to write the prime factorization of 42.

Write two fractions equivalent to each fraction.

20.
$$\frac{1}{3}$$

21.
$$-\frac{15}{24}$$
 22. $-\frac{4}{5}$ **23.** $\frac{16}{28}$

22.
$$-\frac{2}{5}$$

23.
$$\frac{16}{28}$$

Write each fraction in simplest form.

24.
$$\frac{12}{18}$$

25.
$$\frac{27}{54}$$

26.
$$\frac{36}{96}$$

24.
$$\frac{12}{18}$$
 25. $\frac{27}{54}$ **26.** $\frac{36}{96}$ **27.** $\frac{7}{42}$

28. Writing in Math Explain how you can use prime factorization to write a fraction in simplest form.

29. Modeling Draw models to represent
$$\frac{3}{4}$$
 and $2\frac{3}{5}$.

30. What fraction does the shaded part of the model represent?



Compare. Use <, >, or =.

31.
$$\frac{2}{9} = \frac{8}{9}$$

32.
$$\frac{5}{16} = \frac{3}{8}$$

31.
$$\frac{2}{9} = \frac{8}{9}$$
 32. $\frac{5}{16} = \frac{3}{8}$ **33.** $\frac{7}{18} = \frac{2}{5}$

- 34. Ships A crew finds a treasure chest with 168 gold coins and 200 silver coins. Each crew member gets an equal share of each type of coin, with none left over.
 - **a.** What is the greatest possible number of crew members?
 - **b.** How many of each type of coin does each crew member get?

Write as an improper fraction.

35.
$$5\frac{2}{3}$$
 36. $4\frac{5}{6}$ **37.** $8\frac{7}{10}$ **38.** $3\frac{2}{5}$

38.
$$3\frac{2}{5}$$

Write as a whole number or a mixed number.

39.
$$\frac{12}{5}$$
 40. $\frac{30}{9}$ **41.** $\frac{48}{12}$

40.
$$\frac{30}{6}$$

41.
$$\frac{48}{12}$$

42.
$$\frac{42}{30}$$

Write each fraction as a decimal.

43.
$$\frac{2}{16}$$
 44. $\frac{6}{15}$ **45.** $\frac{5}{4}$ **46.** $\frac{8}{25}$

44.
$$\frac{6}{1}$$

46.
$$\frac{8}{25}$$

Write each decimal as a mixed number or fraction in simplest form.

- \bigcirc 51. Mail In one week, $\frac{5}{8}$ of your mail is advertisements and $\frac{1}{5}$ is letters from friends. What is the least number of pieces of mail vou receive?
 - **52.** Order from least to greatest:

$$2.56, -2.\overline{5}, -2\frac{1}{5}, \frac{24}{10}, -2.4$$



Reading Comprehension Read each passage and answer the questions that follow.

Prime Construction Jackie says, "If I multiply the first two prime numbers together and add 1, I get a new prime number." Amit says, "If I multiply the first three prime numbers and add 1, I also get a prime number. Carl says, "I bet the same will happen if I multiply the first four prime numbers and add 1." "It seems to me," says Maria, "that if I multiply any two or more prime numbers, the product is never prime."

- 1. What prime number does Jackie get?
 - **A.** 3
- C. 7

- **B.** 6 **D.** 11
- 2. What prime number does Amit get?
 - **F.** 16

G. 31

H. 41

I. 43

- 3. What number would Carl get?
 - **A.** 107

B. 181

C. 210

- D. 211
- **4.** Which numbers could NOT be used to test Maria's statement?
 - **F.** 3, 13
- **G**. 2, 5, 11
- H. 31, 33
- I. 17, 29

Light Reading Light travels very quickly—at about 1.86×10^5 mi/s. This is fast enough that we do not notice any delay when we flip a light switch. However, light from distant objects in space does not arrive instantaneously. The sun is about 9.3×10^7 mi from Earth. The next nearest star system, Alpha Centauri, is about 2.5×10^{13} mi away.

- 5. About how far away would a lamp have to be for its light to take 2 s to reach our eyes?
 - A. 37,000 mi
 - B. 370,000 mi
 - C. 3.700.000 mi
 - **D**. 37,000,000 mi
- **6.** About how long does it take light from the sun to reach Earth?
 - **F.** 5 s

- **G.** $5 \times 10 \text{ s}$
- H. 5×10^2 s
- I. 500 min

- 7. Which does NOT express the time in seconds it takes light from Alpha Centauri to reach Earth?
 - **A.** $\frac{2.5 \times 10^{13}}{186 \times 10^3}$
- **B.** $\frac{2.5 \times 10^{13}}{186,000}$
- C. $\frac{1.86}{2.5 \times 10^8}$
- **D.** $\frac{2.5}{1.86} \cdot \frac{10^{13}}{10^5}$
- **8.** About how many times farther is Alpha Centauri from Earth than the sun is from Earth?
 - F. 4

- **G**. 23
- **H.** 2.6×10^3
- I. 270,000



Real-World Snapshots



Photographic Memory

Applying Fractions You've probably seen pictures of athletes, animals, or cars that "freeze" the subject's motion but show all the excitement of the moment. A good photographer chooses the best shutter speed for the action. If the shutter stays open too long, the camera records too much movement, and the picture is blurry.





Cameras Then and Now In the 1850s, leather bellows folded a camera into a protective case, making it easier to carry. Zoom lenses on digital

cameras fold into the camera for protection.

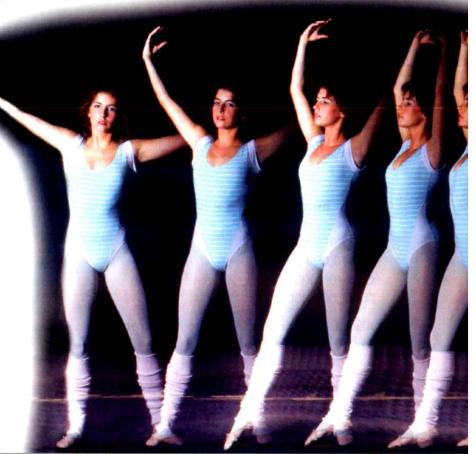
Capturing Movement

For this photograph, the photographer used a very slow shutter speed and a strobe light. The shutter stayed open while the dancer moved.



For this photograph, the photographer used a very slow shutter speed and no flash. The shutter stayed open while traffic moved along the upper and lower levels of Interstate 5 in Seattle, Washington.

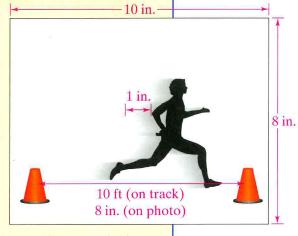


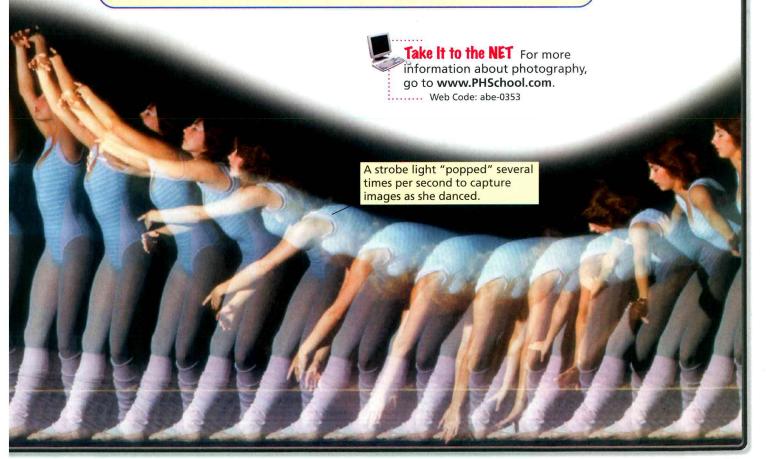


Put It All Together

Data File Use the data on these two pages and on page 129 to answer these questions.

- 1. The image of the runner is blurry because the runner moved a visible amount during the $\frac{1}{15}$ of a second that the shutter was open.
 - **a.** The blur for $\frac{1}{15}$ s is 1 in. long. How long would the blur be for 1 s?
 - **b.** How long would the blur be for each shutter time on page 129?
 - c. Reasoning At what shutter time do you think the length of the blur would be small enough that it would not show in the photo?
- 2. a. How many times would the blur length fit into the space between the two cones in the photo? What fraction of the distance between the cones is the blur?
 - **b.** Use your answer to part (a) to find the distance the runner moved while the shutter was open.
 - **c.** Maintaining the same speed, how far would the runner go in 1 s? In 1 min?







Lessons

- 4-1 Estimating With Fractions and Mixed Numbers
- 4-2 Adding and Subtracting Fractions
- 4-3 Adding and Subtracting Mixed Numbers
- 4-4 Multiplying Fractions and Mixed Numbers
- 4-5 Dividing Fractions and Mixed Numbers
- **4-6** Solving Equations With Fractions
- 4-7 Problem Solving: Try, Check, and Revise and Work Backward
- 4-8 Changing Units in the Customary System
- 4-9 Precision

Key Vocabulary

- benchmark (p. 187)
- precision (p. 227)
- reciprocal (p. 207)

Operations With Fractions

